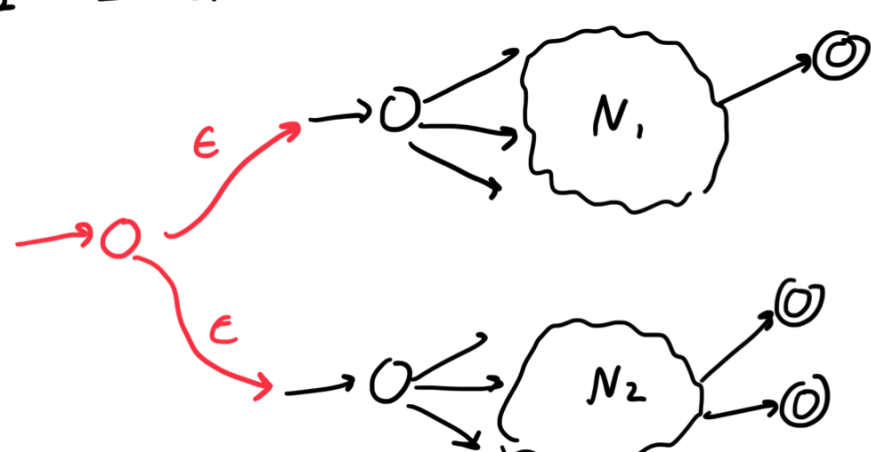


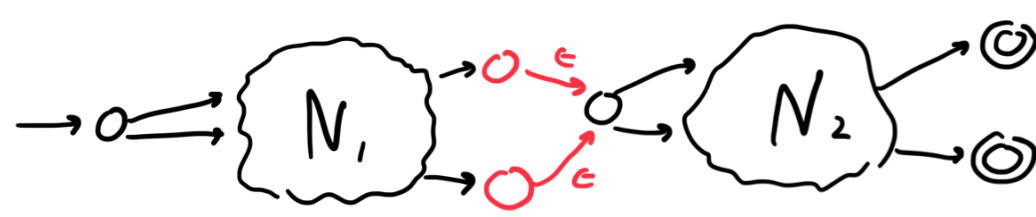
Theorem 1.45 The class of regular languages is closed under union

Proof: Assume A_1 & A_2 are regular
we will construct a machine that recognizes $A_1 \cup A_2$. Let $A_1 = L(N_1)$ and $A_2 = L(N_2)$.

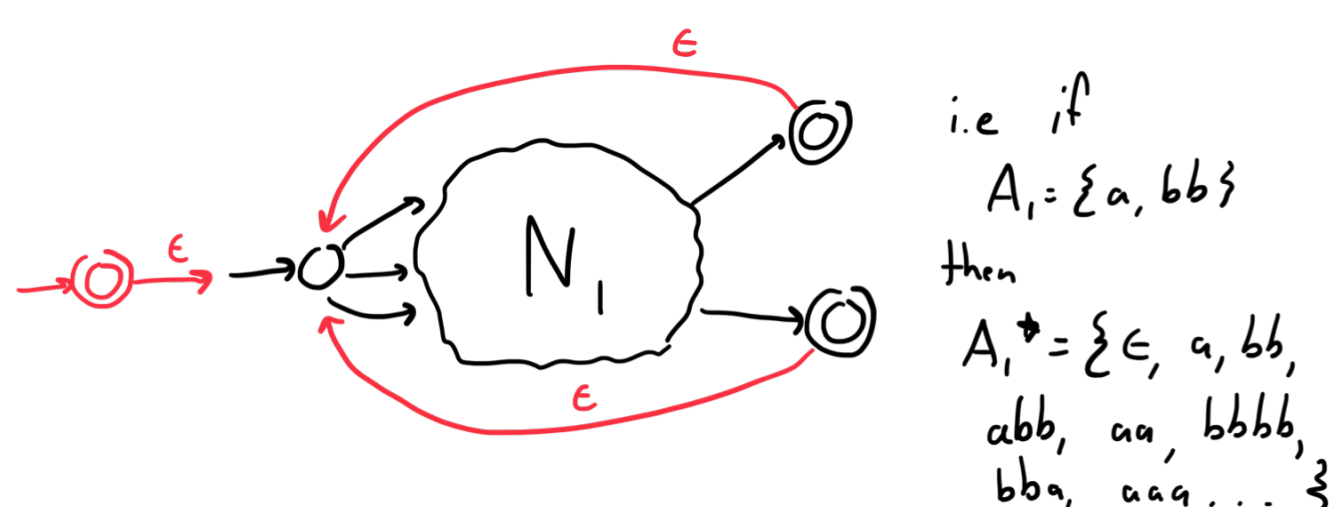


Theorem 1.47 The class of regular languages is closed under union

Proof: Same A_1, A_2, N_1, N_2 as above.

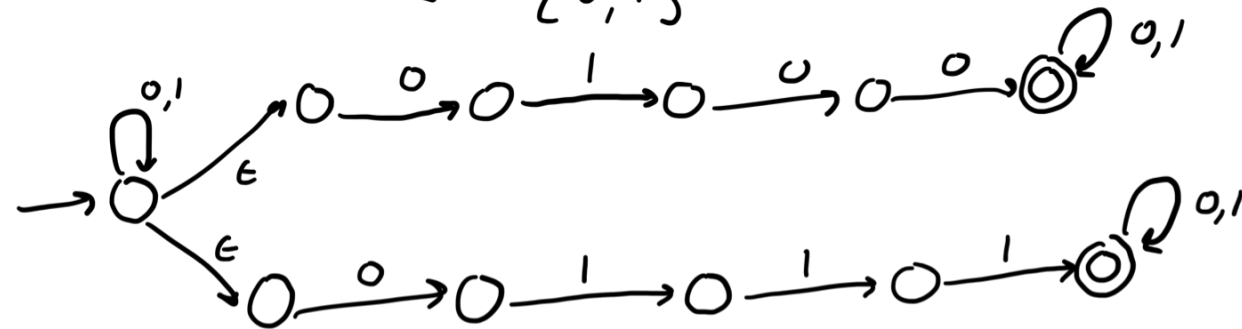


Theorem 1.45 The class of regular languages is closed under star



Another NFA

Let $\Sigma = \{0, 1\}$



Then, $L = \underline{\Sigma^* 01 (00 \cup 11) \Sigma^*}$
regular expression

Regular Expressions - used to describe languages

R is a "regex" if R is:

1. a for some $a \in \Sigma$
2. ϵ
3. $\emptyset$
4. $(R_1 \cup R_2)$ where R_1, R_2 are regex
5. $(R_1 \circ R_2)$
6. (R_1^*)

• $a^+ = a a^*$

• $[a] = a | \epsilon$

• star bounds tightest

$ab^* = a(b^*)$

$\neq (ab)^*$

• concatenation bounds

tighter than union

$aobuc = (ab)uc = ab | c$

$\neq a(buc)$

union/or

Ex: $d \cup ab^*cd^* = (d) \cup (a(b^*)c(d^*))$

Regex $\Leftrightarrow$ Language

a	$\{a\}$
abc	$\{abc\}$
$ab cd$	$\{ab, cd\}$
$a(buc)d$	$\{abd, acd\}$
ab^*c	$\{ac, abc, abbc, abbbc, \dots\}$
$\emptyset$	$\{\}$
$\emptyset^*$	$\{\epsilon\}$
$a(b c)\emptyset$	$\{ab, ac\} \circ \{\} = \{\}$
$a(b \epsilon)c$	$\{abc, ac\}$
$a[b]c$	
$\Sigma = \{0, 1\}$	
$(\Sigma \Sigma \Sigma)^*$	$w: w$ has length a multiple of 3
$\Sigma^* 001 \Sigma^*$	$w: 001$ is a substring in w
$0 \cup 1 \cup 0 \Sigma^* 0 \cup 1 \Sigma^*$	$w: w$ starts & ends with the same symbol

$(a \cup b)^* aba$ to NFA

$a \rightarrow \text{start} \xrightarrow{a} \text{final}$

$b \rightarrow \text{start} \xrightarrow{b} \text{final}$

