

Non-regular Languages

$$B = \{0^n 1^n \mid n \geq 0\}$$

$$C = \{w \mid w \text{ has an equal \# of 0's \& 1's}\}$$

$$D = \{w \mid w \text{ has an equal \# of 01's \& 10's}\}$$

surprise!! D is regular.

How do we prove a language is regular?

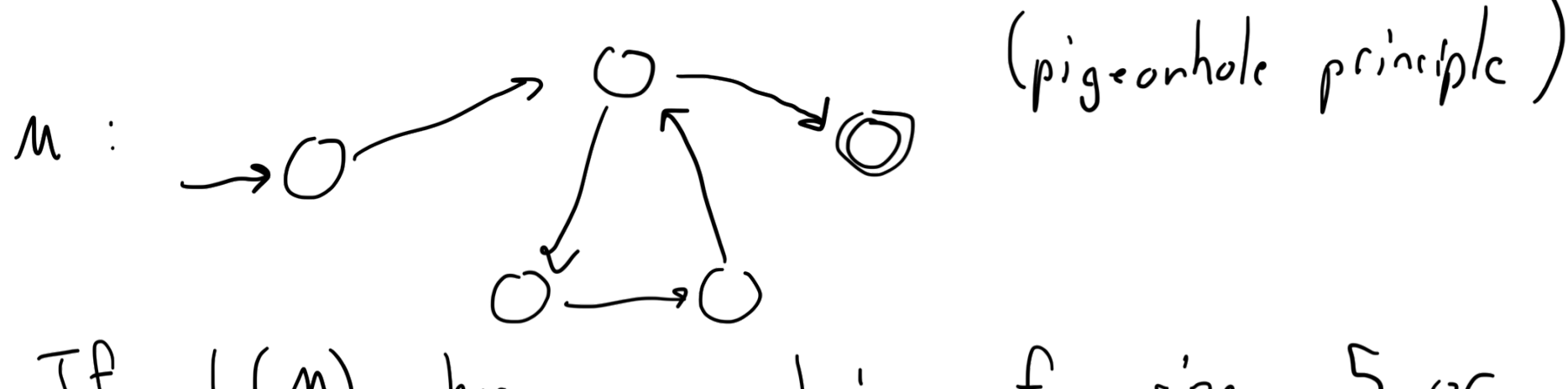
- Draw an NFA/DFA or give a regex.

... Not regular? Cannot verify against all possible NFA's. Instead, use the pumping lemma.

Pumping Lemma

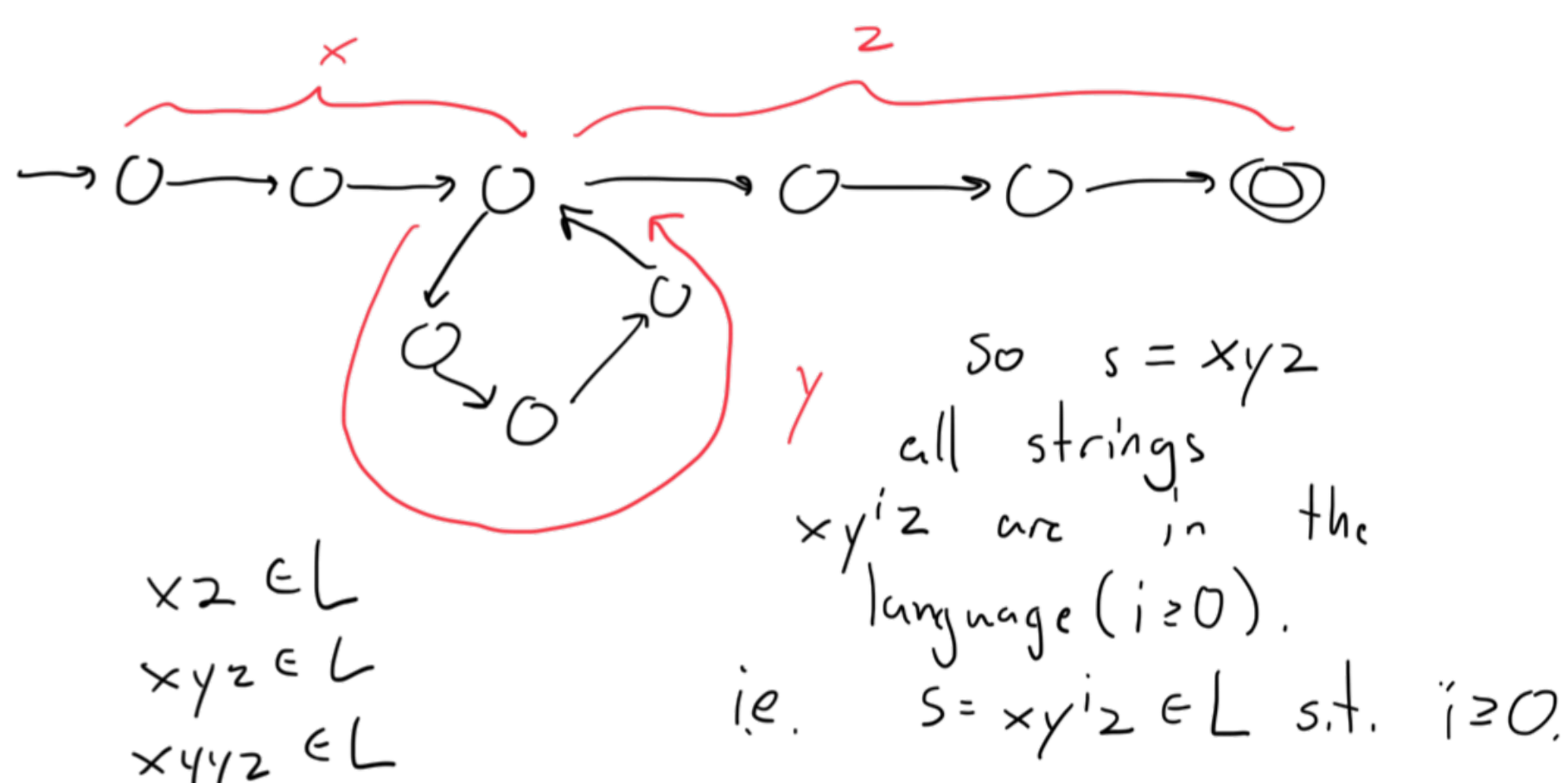
Imagine a FSM that generates long strings

- A FSM has finite states,
- suppose $|Q| = 5$



If $L(M)$ has a string of size 5 or more, it guarantees a cycle in M .

Key idea - if you can go around the cycle once, you can also go around it i times, where $i \geq 0$.



Theorem 1.70

If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p , then s may be divided into 3 pieces, $s = xyz$ satisfying:

- for each $i \geq 0$, $xy^iz \in A$
- $|y| > 0$ ← the cycle has at least edge
- $|xy| \leq p$ ← you have to visit the cycle before the string reaches length p .

Consider a string in an infinitely long language. If the string is too long, then the states we go through must repeat. So, any FSM has a cycle and the "middle" of the string can be pumped.

Let L be regular. If $s \in L$ is sufficiently long, then it can be pumped.

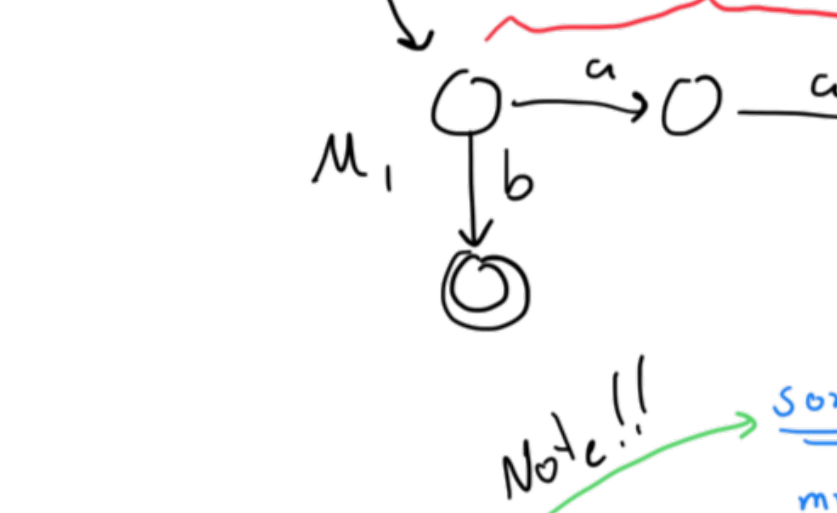
that is, s is longer than the # of states of some FSM that recognizes L .

$$L = \{a, ab, aba\}$$

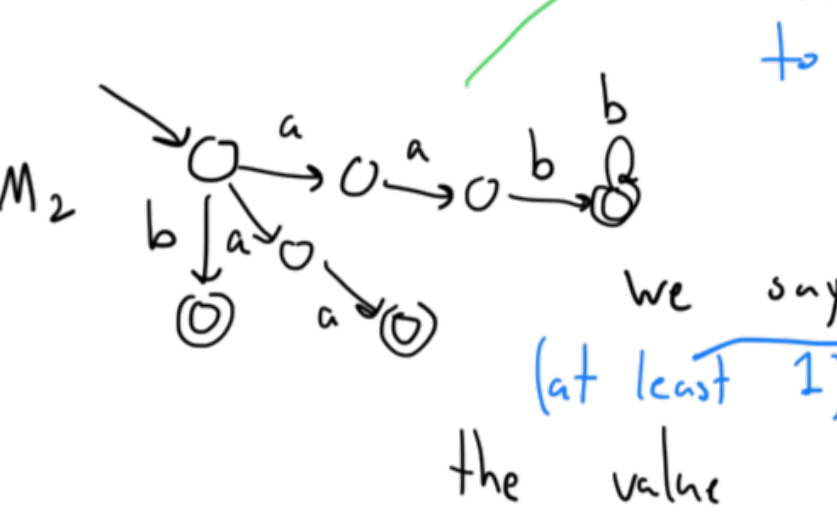


$p = 4$
(p is greater than the size of the longest string in L)
So, Theorem 1.70 holds.

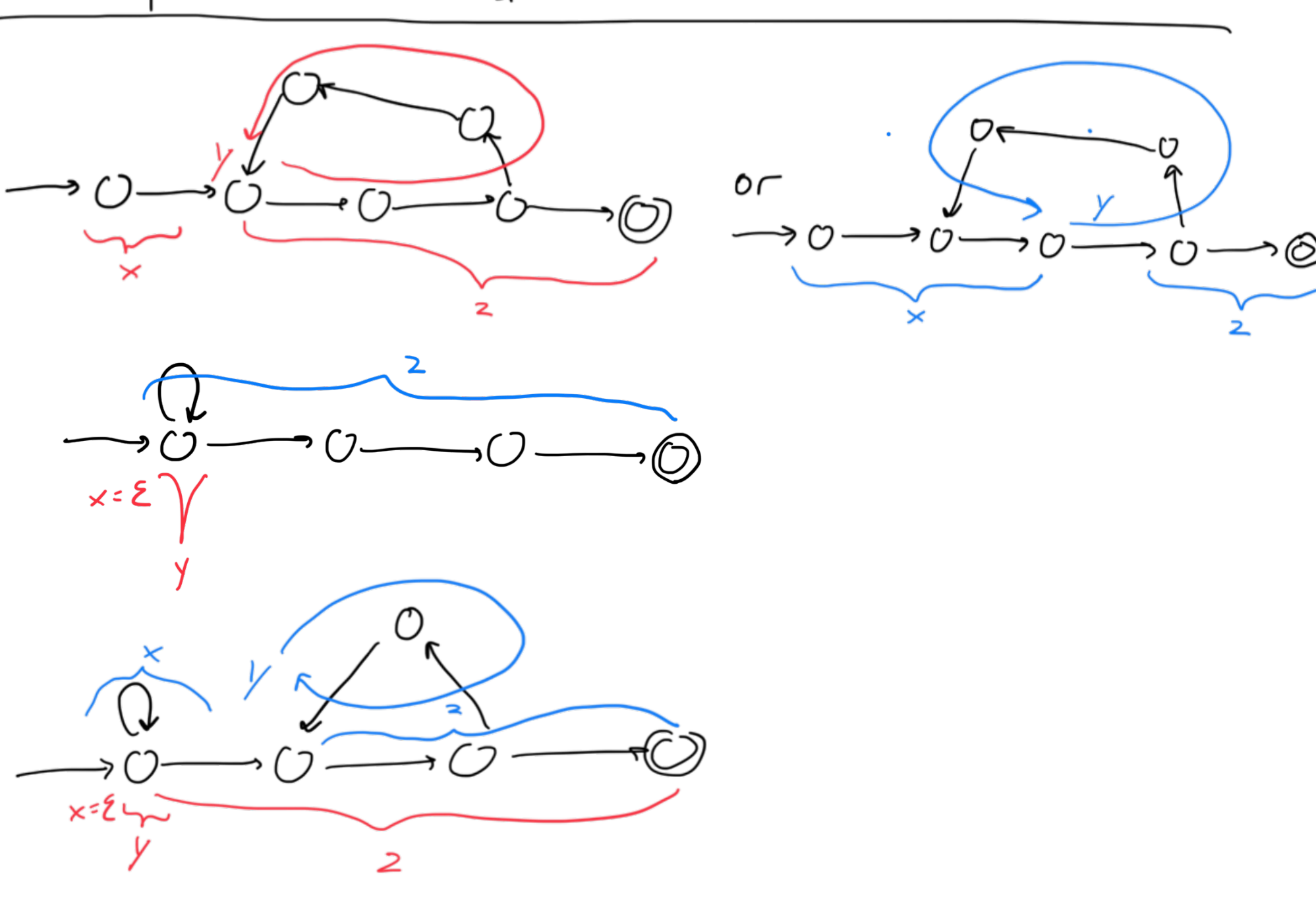
$$L = \{b, aab^*a\}$$



$p = 3$
(any $s \in L$ of size 3 is sufficiently long such that some FSM has a cycle that we must go through for L to be accepted)



Note!! We say "some", not "all".
(at least 1) So, even though $L(M_2) = L$, the value of p is still 2.
ie. p is associated with the language, not its possible machines.



- if L is regular, all strings of length p or greater can be pumped.

We can use the lemma to prove A is not regular. Use contradiction:

- Assume A is regular.
- A has a pumping length p .
- All strings length p or longer can be pumped.
- Find such a string $s \in A$.
- Divide s into xyz .
- Show $xy^iz \notin A$ for some i .
- Consider all cases s can be divided into xyz .
- Show none of these ways satisfy conditions of pumping lemma.
- s cannot be pumped $\Rightarrow$ contradiction
- You're done!

Ex 1

$B = \{0^n 1^n \mid n \geq 0\}$ Prove B is not regular. Proof. Suppose not. Thus, B is regular.

So, B has a pumping length p .

$$\text{Let } s = 0^p 1^p$$

must have length p or more

Let's divide s into pieces xyz .

ie. $p = 3$

Case 1. y contains only 0's or only 1's.

So, $xy^iz \in B$ for any $i \geq 0$.

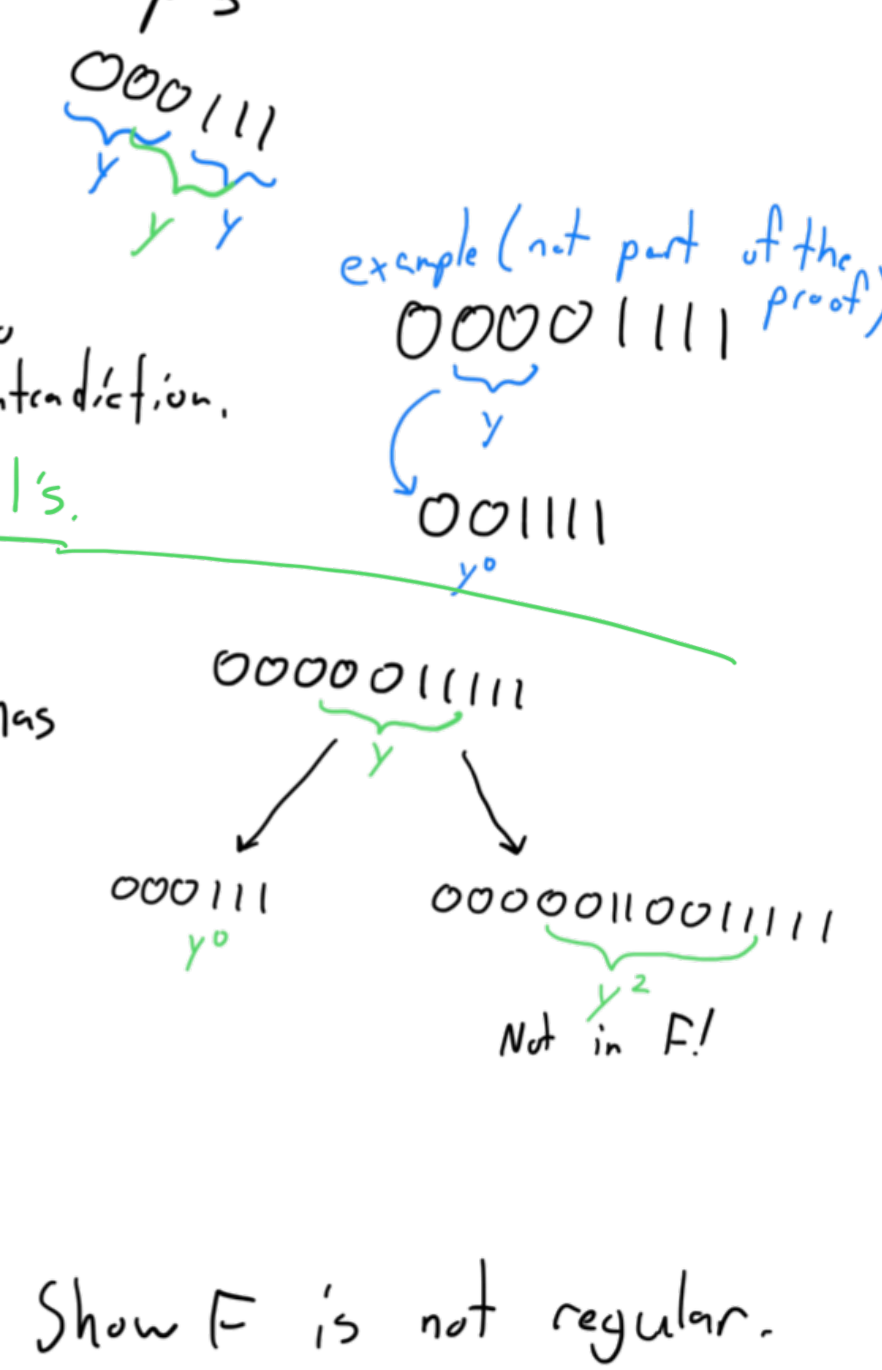
But, $xy^2z \notin B$ because it has too few 0's (respectively 1's). Contradiction.

Case 2. y contains both 0's & 1's.

So, $xy^iz \in B$ for any $i \geq 0$.

But, $xy^2z \notin B$ because it has 0's & 1's out of order, a contradiction.

□



Ex 2

$F = \{ww \mid w \in \{0,1\}^*\}$ Show F is not regular.

Proof. Assume F is regular. Let p be the pumping length.

$$\text{Let } s = 0^p 1 0^p$$

As an example, if $p = 3$ then $0^3 1 0^3 = 00010001$

Recall $|y| > 0$ and $|xy| \leq p$ by the pumping lemma.

Therefore, y contains all 0's.

We have $xy^iz \in F$ for any $i \geq 0$, but $xy^2z \notin F$, a contradiction. □

Regular "World"

