

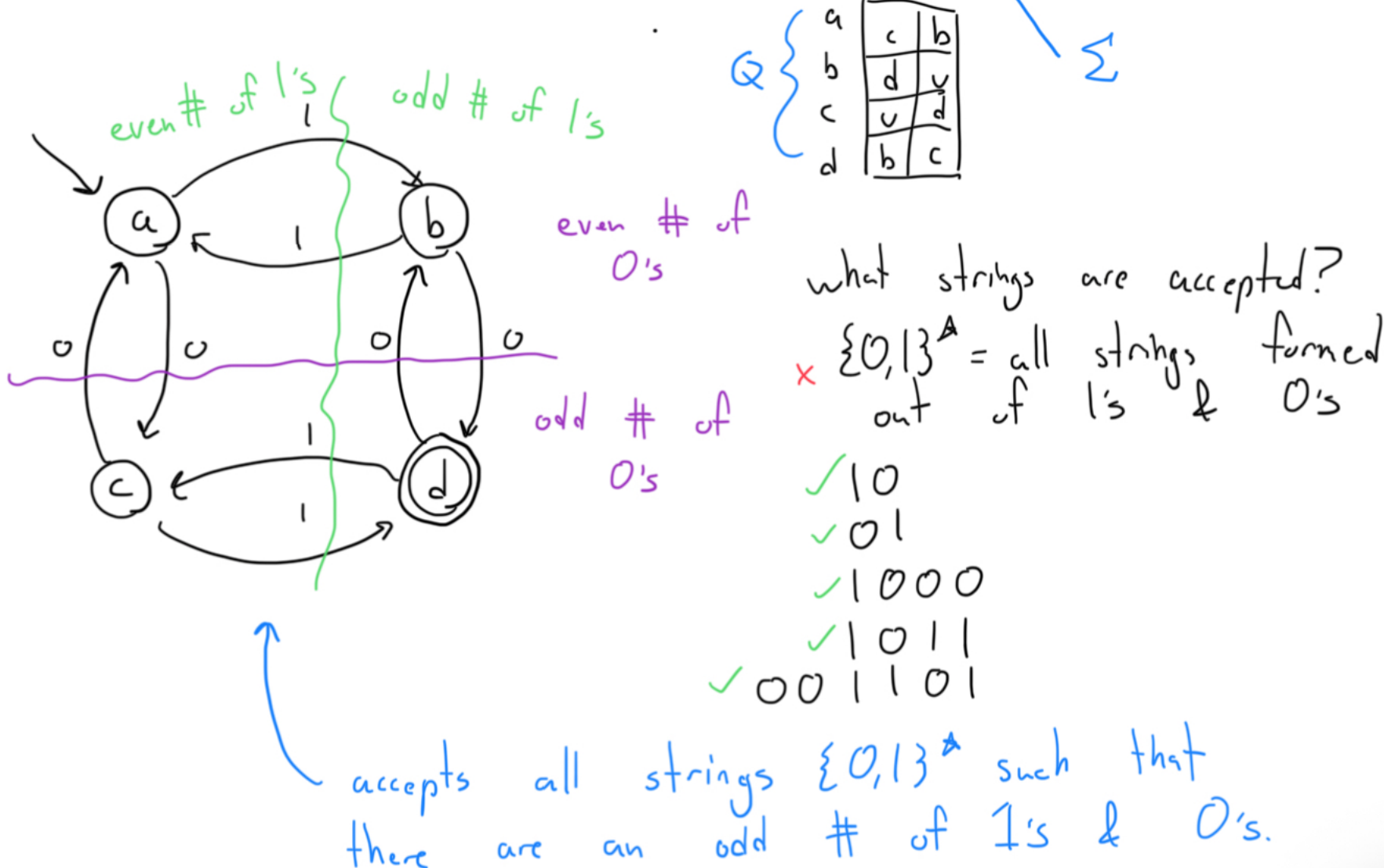
Ch 1 Finite State Machines

- FSM for short, or "finite automaton" (a) ← plural!
- simplest model of computation. Has finite, limited memory

Formal Definition A deterministic, finite automaton M is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$.

- Q : finite set of states
- Σ : alphabet
- δ : transition function $\delta: Q \times \Sigma \rightarrow Q$
- $q_0 \in Q$: start (initial) state
- $F \subseteq Q$: set of final (accepting) state(s)

Ex 1 Let $Q = \{a, b, c, d\}$, $\Sigma = \{0, 1\}$, $q_0 = a$
 $F = \{d\}$ and $\delta =$



Ex 2



$\Sigma = \{25\}$
 $Q = \{q_0, \dots, q_3\}$
 $F = \{q_3\}$
 $\delta = \delta(q_0, 25) = q_1$
 $\delta(q_1, 25) = q_2$
 $\delta(q_2, 25) = q_3$
 $\delta(q_3, 25) = q_3$

Simulates a washing machine that accepts quarters.

Notes every vertex, or node, must have an outgoing edge for every symbol.

• FSM accepts the ϵ string if the initial state is a final state

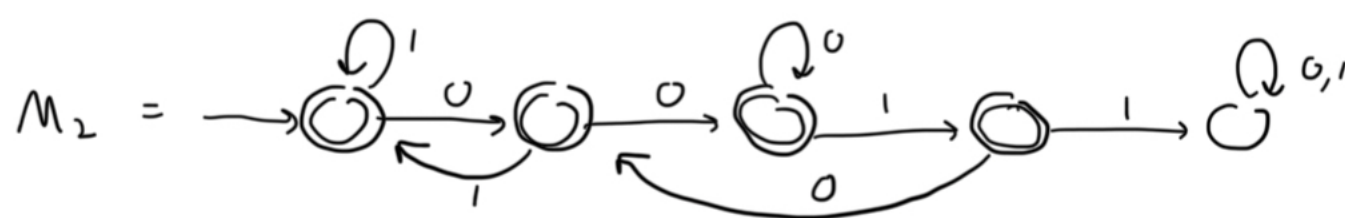
• FSM recognizes the empty language $\emptyset$ if there is no walk to any final state



Design Example



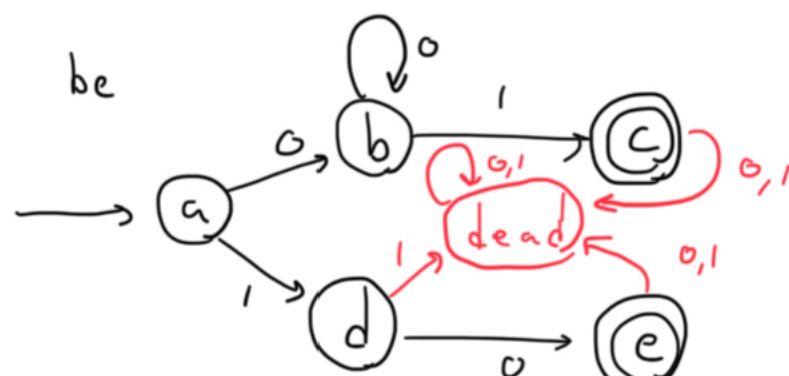
$L(M_1)$ = the language of M_1 = the set of strings that contain 0011 as a substring (over $\{0, 1\}^*$).



$L(M_2)$ = the set of strings over $\{0, 1\}^*$ that do not contain 0011 as a substring.

Notice that $L(M_1) = \overline{L(M_2)}$

Let M be



then $L(M)$ is either 10

or $01, 001, 0001, \dots, 0^+1$

$L(M) = \{w \mid w \text{ is either } 10 \text{ or a string of at least one } 0 \text{ followed by a } 1\}$.

Def A language is regular iff some FSM recognizes it.

What languages are NOT regular?

- anything that requires memory
- ↳ an FSM cannot store strings
- ↳ cannot count

i.e. $ww \rightarrow 0110101101$
 $w = 01101$

or $0^n 1^n \rightarrow 000000111111$
 $n = 6$

Regular Operations

Union

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Concatenation

$$A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$$

Star

$$A^* = \{x_1 x_2 \dots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$$

Ex: let $A = \{aa, b\}$ and $B = \{x, yy\}$,

then $A \cup B = \{aa, b, x, yy\}$

$A \circ B = \{aax, aayy, bx, byy\}$

$A^* = \{\epsilon, aa, b, aaaa, aab, baa, bb, aaaaaa, aaaaab, aabaa, aabb, \dots\}$