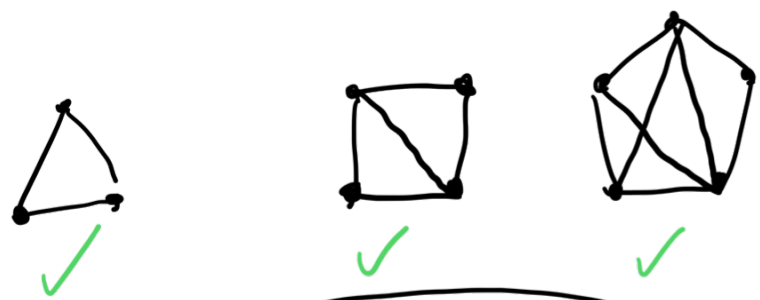
Chordal Graphs

a graph is chordal if every cycle of size 4 or more has a chord (a cross-edge).





Let HALF-CLQ = {⟨G⟩ | G is a simple, finite graph having a clique subgraph with at least n/2 nodes, where |V| = n.

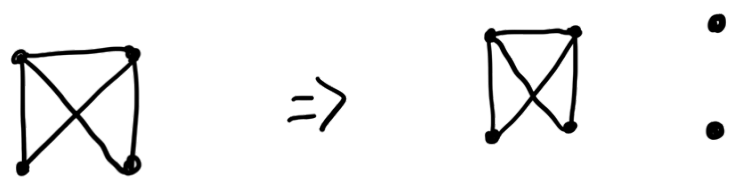


Theorem 7.23. HALF-CLQ is NPC.

Proof: First, we show it is in NP. Given a graph G and a subgraph S ⊆ G, it is easy to check whether S is a half-clique by iterating through the vertices of S to see if they are all adjacent (in O(n)) and then checking that |S| ≥ |V|/2.

Next, we show it is NP-Hard via a reduction from CLIQUE. In CLIQUE, we are given k and a graph G and asked if G has a k-clique.

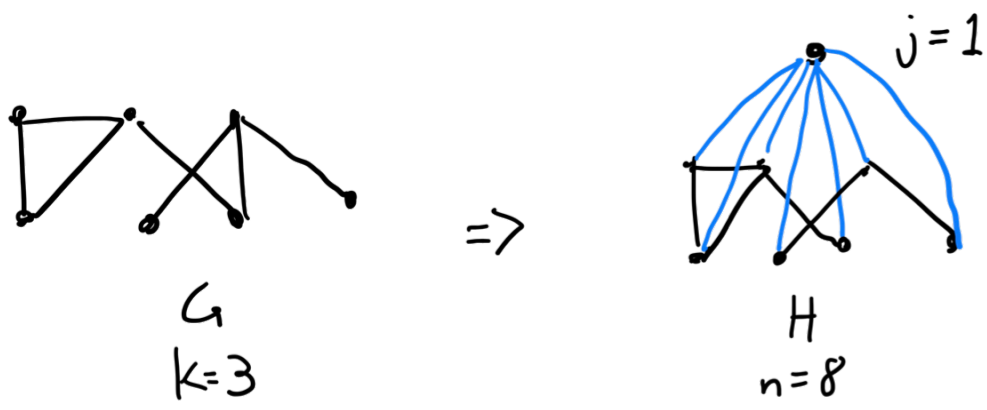
Case 1



If k ≥ n/2, we add j nodes (an indep. set) to form graph H s.t. H has 2k vertices.

Clearly, H has a clique of size n/2 iff G has a clique of size k.

case 2



If k < n/2, we add j nodes adjacent to all of G & to one another. s.t. j = n − 2k.

⇒ Let G have a clique of size k. Any arbitrary clique in G combines with the j new nodes to make a larger clique. Thus, H has a clique of size exactly n/2 (a half-clique).

⇐ Let H have a half-clique. Clearly, H has a k-clique because k < n/2. Thus, G has a k-clique.

So, HALF-CLQ is NP-Hard ∩ NP. It is NPC.