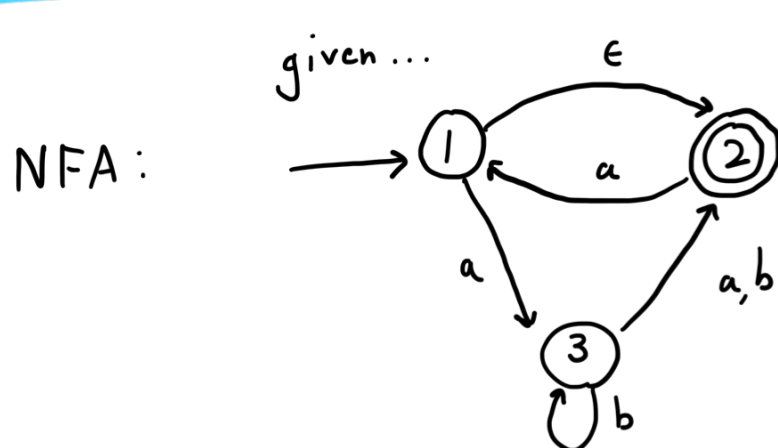
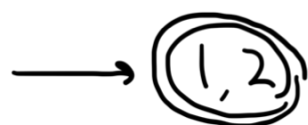


NFA's with ϵ -transitions



DFA: the start state is



Another "Pumping Lemma Proof"

Let $A = \{0^i 1^j, i \geq j\}$. Prove that A is not regular.

Proof: Suppose A is regular. Let p be the pumping length. Choose $s = 0^{p+1} 1^p$.

We can divide s into xyz s.t. $|y| > 0$ and $|xy| \leq p$ and $xy^k z \in A$ for some $k \geq 0$, so y is only 0's.

ex. $p=3$ gives

0000111

ex. y then

0000111

00000111

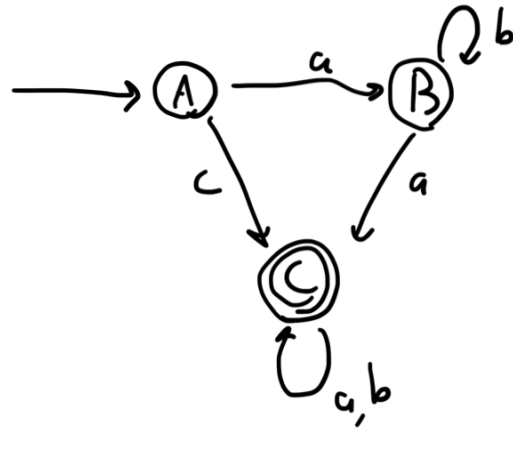
00111

But, $xy^0 z = xz \notin A$ since it has too few 0's, a contradiction.

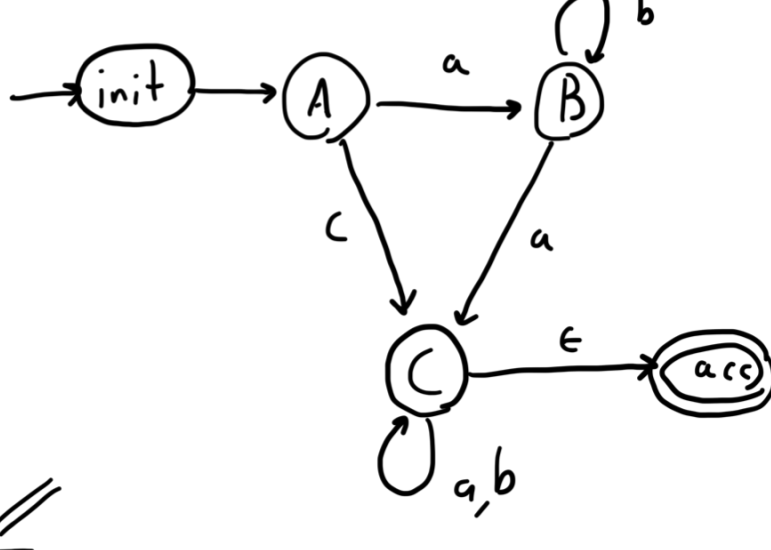
Another "NFA to Regex" construction

NFA

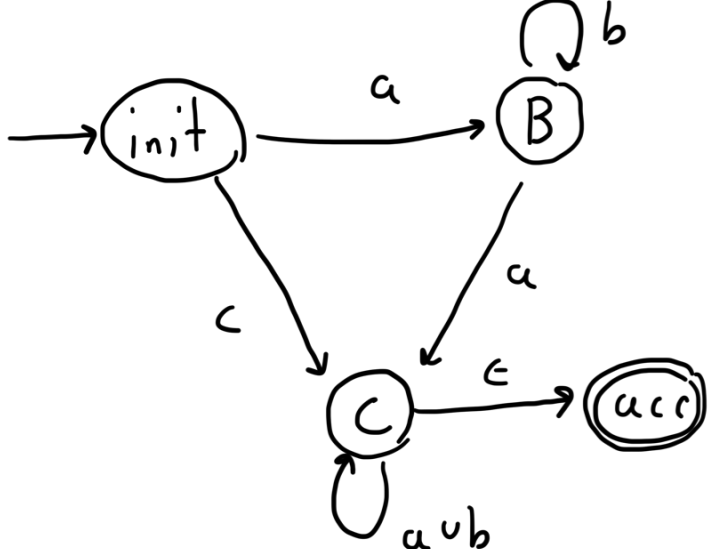
GNFA



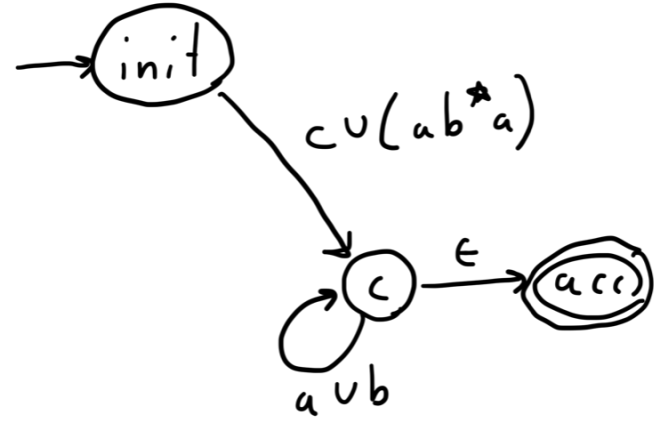
$\Rightarrow$



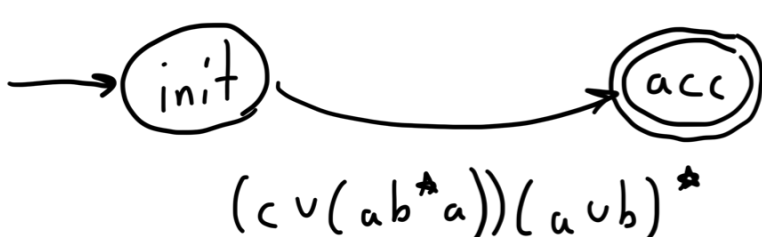
$\Leftarrow$



$\Rightarrow$



$\Leftarrow$



FSM's cannot handle powers

$L = a^{2^n} = \{a, aa, aaaa, aaaaaa, \dots\}$

Prove that L is not regular.

Proof: Suppose L is regular. It has pumping length p . Choose $s = a^{2^p}$.

We can split $s = xyz$ where $xy^i z \in L$ ($i \geq 0$) by the pumping lemma. Since $|xy| \leq p$, y is made of p or fewer a 's.

See that $xy^2 z \notin L$ because we cannot reach the next power of 2 by pumping y up once.

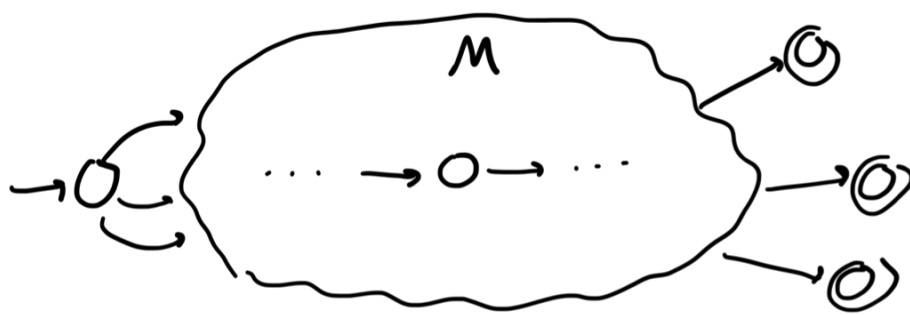
Ex. $aaaaa \in L \Rightarrow aaaaaa \notin L$

We have a contradiction. $\square$

Regular languages are closed under reversal.

If L is a regular language, then $L^R = \{w^R : w \in L\}$ is regular.

Proof: Let M be an NFA that recognizes L .

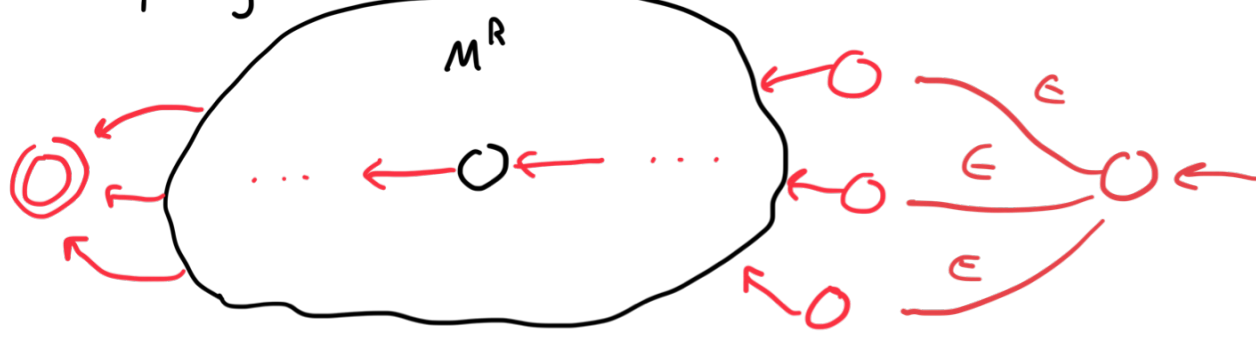


1. Reverse every transition in M .

$\xrightarrow{a} O \xrightarrow{b} \Rightarrow \xleftarrow{a} O \xleftarrow{b}$

2. Make the old start state an accept state

3. Create a new start state, connected to the old accepting states via ϵ -edges.



M^R clearly recognizes L^R . $\square$