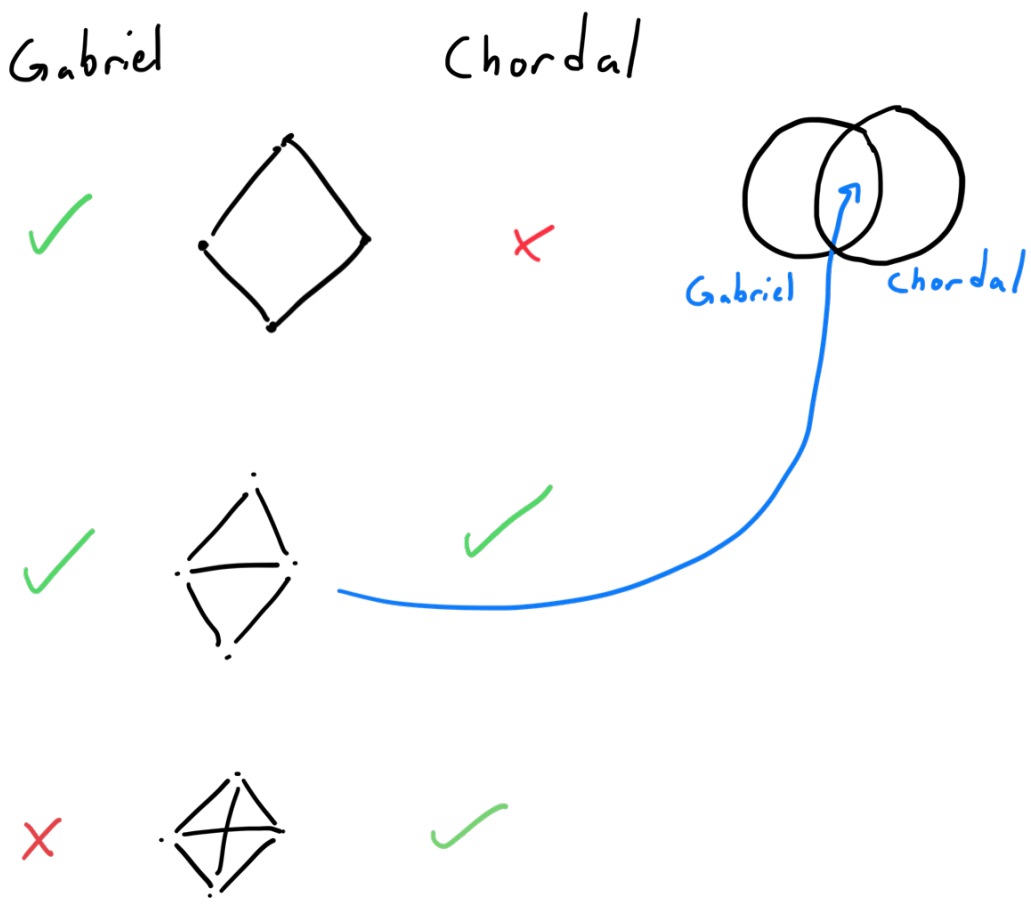
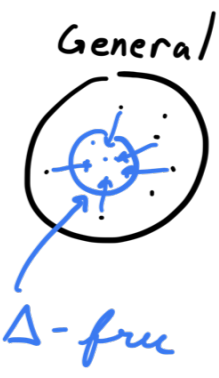
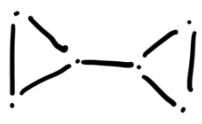


Hamiltonian Cycle (HC) is NPC even on triangle-free graphs (Δ-free)

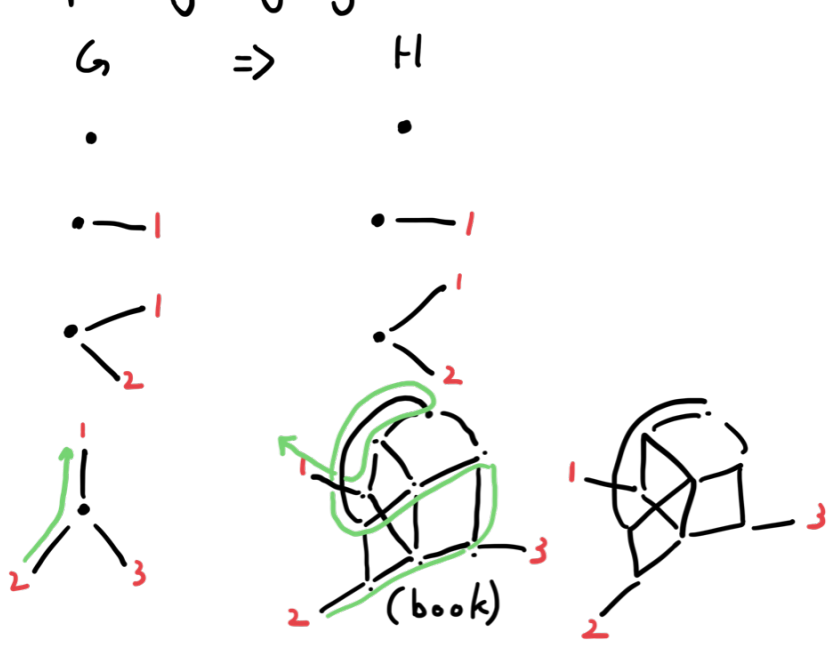


Proof: NP is obvious.
we will prove it is NP-hard via reduction from HC on maximum degree 3 graphs. (is forbidden)



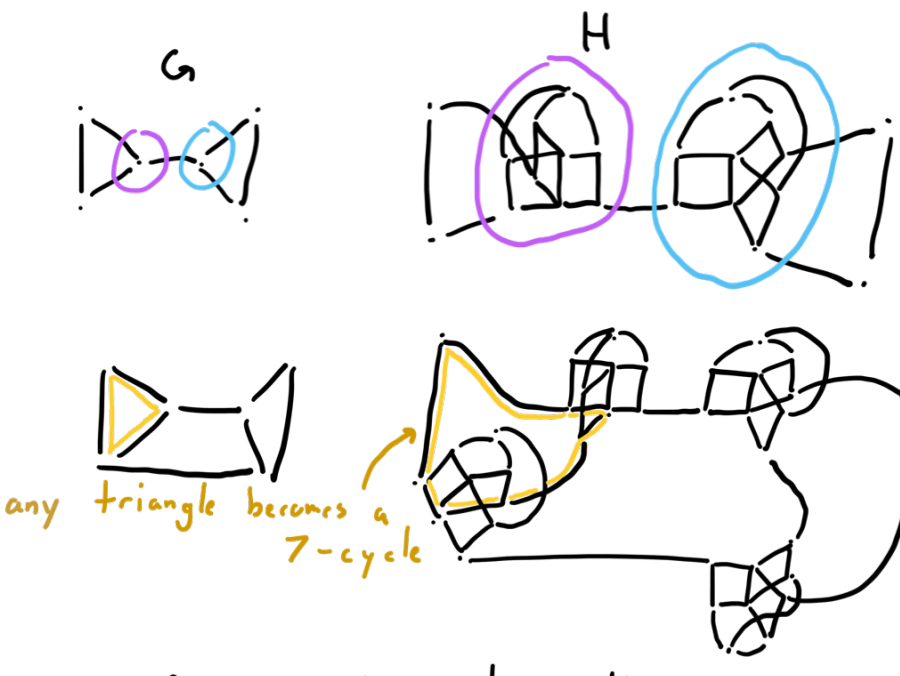
G

Given G, we construct a triangle-free graph H by converting each vertex of distinct degree into a corresponding gadget.



Iterating all degree 3 vertices & converting them into "books" takes $O(n+|V|)$.

The degree-3 gadget is Δ-free & it simulates the fact that HC is G cannot use all 3 edges incident on any vertex of degree 3



If a path enters the degree-3 gadget in H, it must visit every vertex & leave through one of the bridge edges, otherwise no HC will be found.

Also, H expands the size of every cycle in G, & no gadget has a Δ, so H is Δ-free.

Thus, any HC in G exists iff a HC in H exists. □