

The Dynamic Programming Algorithm

The CYK Algorithm

by

Cocke, Younger (1967), Kasami (1965)

This presentation is designed to be used to understand the CYK algorithm relative to Example 3.6.1 on pages 156-157 of Lewis and Papadimitriou's book, *Elements of the Theory of Computation*, 2/e (1998).

The Dynamic Programming Algorithm

- Given CFG $G=(V,S,R,S)$ where
 $V = \{S\} \dot{\cup} S$ $S = \{ (,) \}$
 $R:$ $S \circlearrowleft SS$
 $S \circlearrowleft (S)$
 $S \circlearrowleft e$

- $G=(V,S,R,S)$ in *Chomsky Normal Form* (CNF) becomes

$$V = \{S\} \dot{\cup} S \quad S = \{ (,) \}$$

$$R: \quad S \circlearrowleft SS$$

$$S \circlearrowleft (S_1)$$

$$S_1 \circlearrowleft S$$

$$S \circlearrowleft ()$$

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

$$R: \quad S \circlearrowleft SS$$

$$S \circlearrowleft (S_1)$$

$$S_1 \circlearrowleft S$$

$$S \circlearrowleft ()$$

$$N_{i,i+s} = \{ A \in V : A \Rightarrow^* x_i \dots x_{i+s} \}$$

$$\forall i, s \exists 1 \leq i \leq i+s \leq n.$$

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

$$R: \quad S \circlearrowleft SS$$

$$S \circlearrowleft (S_1)$$

$$S_1 \circlearrowleft S$$

$$S \circlearrowleft ()$$

	1	2	3	4	5	6	7	8
1	(
2		(
3)					
4				(
5					(
6)		
7)	
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

$$R: \quad S \circlearrowleft SS$$

$$S \circlearrowleft (S_1)$$

$$S_1 \circlearrowleft S$$

$$S \circlearrowleft ()$$

	1	2	3	4	5	6	7	8
1	(ϵ						
2		(
3)					
4				(
5					(
6)		
7)	
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

$$R: \quad S \circlearrowleft SS$$

$$S \circlearrowleft (S_1)$$

$$S_1 \circlearrowleft S$$

$$S \circlearrowleft ()$$

	1	2	3	4	5	6	7	8
1	(ϵ						
2		(S					
3)					
4				(
5					(
6)		
7)	
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ						
2		(S					
3)	Æ				
4				(Æ			
5					(
6)		
7)	
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ						
2		(S					
3)	Æ				
4				(Æ			
5					(S		
6)		
7)	
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ						
2		(S					
3)	Æ				
4				(Æ			
5					(S		
6)	Æ	
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ						
2		(S					
3)	Æ				
4				(Æ			
5					(S		
6)	Æ	
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ					
2		(S	Æ				
3)	Æ	Æ			
4				(Æ	Æ		
5					(S		
6)	Æ	
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3/2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ					
2		(S	Æ				
3)	Æ	Æ			
4				(Æ	Æ		
5					(S	S_1	
6)	Æ	
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ					
2		(S	Æ				
3)	Æ	Æ			
4				(Æ	Æ		
5					(S	S ₁	
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ					
2		(S	Æ				
3)	Æ	Æ			
4				(Æ	Æ		
5					(S	S ₁	
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ				
2		(S	Æ	Æ			
3)	Æ	Æ	Æ		
4				(Æ	Æ		
5					(S	S ₁	
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ				
2		(S	Æ	Æ			
3)	Æ	Æ	Æ		
4				(Æ	Æ	S	
5					(S	S ₁	
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ				
2		(S	Æ	Æ			
3)	Æ	Æ	Æ		
4				(Æ	Æ	S	
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ				
2		(S	Æ	Æ			
3)	Æ	Æ	Æ		
4				(Æ	Æ	S	
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ			
2		(S	Æ	Æ	Æ		
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ			
2		(S	Æ	Æ	Æ		
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ			
2		(S	Æ	Æ	Æ		
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ		
2		(S	Æ	Æ	Æ		
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ		
2		(S	Æ	Æ	Æ	S	
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ		
2		(S	Æ	Æ	Æ	S	
3)	Æ	Æ	Æ	Æ	
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	
2		(S	Æ	Æ	Æ	S	
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	
2		(S	Æ	Æ	Æ	S	
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	
2		(S	Æ	Æ	Æ	S	S ₁
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	
2		(S	Æ	Æ	Æ	S	S ₁
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	S
2		(S	Æ	Æ	Æ	S	S ₁
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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The Dynamic Programming Algorithm

- Can $w = x_1 x_2 \dots x_n$, $n^3 2$, be generated by G ?
- Consider the string $w = (((())))$

	1	2	3	4	5	6	7	8
1	(Æ	Æ	Æ	Æ	Æ	Æ	S
2		(S	Æ	Æ	Æ	S	S ₁
3)	Æ	Æ	Æ	Æ	Æ
4				(Æ	Æ	S	S ₁
5					(S	S ₁	Æ
6)	Æ	Æ
7)	Æ
8)

N

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