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Robust fuzzy logic control of mechanical systems

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Abstract

An approach for the design of fuzzy control laws for tracking control of a large class of mechanical systems is proposed. The approach employs the framework of Lyapunov's stability theory to formulate a class of control laws that guarantee convergence of the tracking errors to within specification limits in presence of bounded parameter uncertainties and input disturbances. The proposed control laws possess a large number of parameters and functional relationships to be chosen by the designer according to a methodology developed in the paper. The large number of design degrees of freedom makes the approach suitable for fuzzy logic implementation. A number of fuzzy implementations of the proposed control methodology are provided. All implementations guarantee tracking error convergence to within prespecified performance limits. An extensive simulation study using a model of a two-degree-of-freedom robot manipulator was conducted. Fuzzy and non-fuzzy implementations of the proposed methodology were compared to control laws designed using other design methods. Simulation study results indicate a superiority of the proposed control methodology compared to other approaches. The study also demonstrates better performance of the fuzzy control implementation compared to its non-fuzzy counterpart. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy control; Mechanical systems; Robotics

1. Introduction

Since its inception by Zadeh [36], Fuzzy Logic Theory has attracted diehard supporters and antagonists. In the context of control engineering, the former group advocates the universal aptness of Fuzzy Logic Theory to all control problems based on its successful application to consumer appliances and industrial processes [35]. This success is due to the Fuzzy Logic systems' ability to provide explicit model-free control of complex plants (since the rules necessary for fuzzy operation can be provided by an expert or an statistical system, such as Neural Networks, from training data),

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their ability to adapt to conditions different from design, and the parallelization of their computations. The diehard antagonists of Fuzzy Logic Control (FLC) claim that conventional control based on rigorous mathematics can outperform any FLC. However, in the last decade a third point of view has emerged which seeks consensus from a synthesis of both methodologies.

The emerging hybrid control methodology combines the conventional control theory with soft-computing theories to create controllers for dynamical systems [10]. An increasing amount of research on integration of qualitative knowledge (for example, pilot experience) and quantitative knowledge (analytical models) through fuzzy modeling and control has been applied to flight control laws for helicopters [25, 12, 21], aircraft and spacecraft [9, 20] (see survey in [22]).

Mechanical systems with large uncertainties, non-linearities and bounded external disturbances have been successfully controlled using Lyapunov-based robust controllers (see the survey in [15]). In the past few years, the major focus of conventional mechanical systems research has been on robust and adaptive controllers [18, 29, 30, 3] where a nominal model is assumed and uncertainty accounts for all aspects of the system not accounted for by this nominal model. Some properties of the nominal system and uncertainty are assumed, such as the boundedness of the inertia matrix, the structure of the bound of the Coriolis/centrifugal forces, and the boundedness of the gravitational forces. Many such controllers have been proposed in the last decade, for which the tracking errors converge to a given manifold. In [18], a proportional-derivative (PD) plus robust non-linear decentralized (decoupled) control law with constant coefficients is proposed such that the error is globally convergent to a residual set around a linear manifold. A similar controller is proposed in [29], and is later extended to a non-linear manifold in [30] by requiring exact knowledge of the nominal dynamic equations. In [3], a robust controller also requiring knowledge of all the nominal dynamic equations is designed such that the error converges to a linear manifold and the closed-loop system is uniformly ultimately bounded. All these controllers have a “boundary layer” around the error manifold, where the control effort obeys different rules in order to avoid chattering.

Considerable research effort has been directed toward laying the foundation of a systematic design methodology of FLC systems [34]. In the last decade, there have been several research directions in Fuzzy Logic Control. One deals with modeling and stability of Takagi–Sugeno–Kang (TSK) linear-time-varying type systems using Lyapunov Linear Matrix Inequalities and H_∞ methodologies [28, 27, 16, 24, 11]. The corresponding control strategy is closely related to traditional gain scheduling. Another research direction deals with stability of PID-like Fuzzy Logic Controllers (PID-FLCs) [8, 33, 4] based on classical control theory techniques. Most methodologies used for the design of membership functions are either intuitive or rely on finding local equivalent PID gains for the PID-FLC. A third research direction focuses on the application of Lyapunov, variable structure (Sliding Mode) and adaptive control techniques to non-linear systems for which the nominal dynamics (analytic functions or TSK models) and the bounds on the model uncertainties are known [31, 19, 2, 26]. In a Sliding Mode Controller, application of Lyapunov’s Stability Theory guarantees that the tracking error remains close to the error manifold (sliding surface).

This paper presents a new robust control design approach for tracking control of mechanical systems. The approach is based on Lyapunov Stability Theory and extends the work of Corless and Leitmann [6] on robust control of nonlinear systems. It also extends the work of Efrati and Flashner [7]. The proposed control law defines a large class of robust tracking controllers allowing a great deal of flexibility for the designer, thus making it appropriate for Fuzzy Logic implementation to enhance the overall system performance. The importance of the new methodology is that it provides

a general procedure for the design of robust–fuzzy hybrid (analog+fuzzy) control laws for a large class of mechanical systems.

The paper is organized as follows. In Section 2, the characteristics of mechanical systems and Input-to-State definition and theorem are presented. The Fuzzy Logic Controller (FLC) is presented in Section 3. The new robust tracking law is formulated in Section 4, and the performance of the proposed approach is demonstrated in a simulation example for a rotational–prismatic planar robot arm in Section 5. Concluding remarks are given in Section 6.

2. Characteristics of mechanical systems

Consider the motion of an n -degrees-of-freedom mechanical system defined by the generalized coordinates $q \in R^n$. The equation of motion can be written as

$$M(q)\ddot{q} + C(q, \dot{q}) + g(q) = Q + \tau_d, \quad (1)$$

where $M(q) \in R^{n \times n}$ is the symmetric positive-definite inertia matrix $\forall q$, the inertial terms are represented by $M(q)\ddot{q}$, the Coriolis (centripetal) terms are represented by $C(q, \dot{q}) = \dot{M}(q)\dot{q} - \frac{1}{2}(\partial/\partial q)(\dot{q}^T M(q)\dot{q})$, the potential terms are represented by $g(q)$, Q is a control law and τ_d represents the bounded disturbances.

The system described by Eq. (1) has the following properties:

1. The inertia matrix $M(q)$ is bounded:

$$\lambda_m I \leq M(q) \leq \lambda_M I, \quad (2)$$

where λ_m ($\lambda_M < \infty$) denotes the strictly positive minimum (maximum) eigenvalue of M for all configurations of q .

2. The Coriolis and centrifugal terms $C(q, \dot{q})$ satisfy

$$\|C(q, \dot{q})\| \leq v_b^*(q) \|\dot{q}\|^2, \quad (3)$$

where $v_b^*(q)$ is a known scalar function. Since q is bounded for most practical engineering applications, $v_b^*(q) \leq v_b$ for a constant $v_b > 0$.

3. It is always possible to find $\bar{C}(q, \dot{q})$ such that

$$C(q, \dot{q}) = \bar{C}(q, \dot{q})\dot{q} \quad (4)$$

and $S = \frac{1}{2}\dot{M}(q) - \bar{C}(q, \dot{q})$ is skew-symmetric, i.e.

$$z^T S(q, \dot{q}) z = 0 \quad \forall z \in R^n. \quad (5)$$

4. The gravity vector $g(q)$ satisfies

$$\|g(q)\| \leq g_0^*(q), \quad (6)$$

where $g_0^*(q)$ is known.

5. The system dynamics equation (1) can always be written as

$$Y(q, \dot{q}, \ddot{q})\varrho = Q, \quad (7)$$

where $Y(q, \dot{q}, \ddot{q}) \in R^{n \times r}$ and $\varrho \in R^r$ is a vector of dynamic parameters.

6. It is also assumed that the disturbances are bounded as follows:

$$\|\tau_d\| \leq \Theta. \quad (8)$$

3. Structure of the proposed fuzzy logic controller

In this section, relevant terminology and results are presented. For a more detailed discussion see [34].

The fuzzy membership functions are chosen to satisfy the normality requirement. The triangular and Gaussian membership functions are used in this work, defined as

$$\mu_{A^l}(x) = \begin{cases} 1 - |x - \underline{x}^l|/b_x & \text{if } |x - \underline{x}^l| < b_x, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

$$\mu_{A^l}(x) = e^{-((x - \underline{x}^l)/a_x)^2}, \quad (10)$$

where $l = 1, 2, \dots, N$ and the centers of the membership functions satisfy $\underline{x}^l < \underline{x}^{l+1}$. The advantage of Gaussian membership functions is that they are continuously differentiable for all x .

The Fuzzy Logic Controller structure used has product inference engine, singleton fuzzifier and center average defuzzifier, with the following form (see [34]):

$$f(x) = \frac{\sum_{l=1}^M \bar{y}^l (\prod_{i=1}^n \mu_{A_i^l}(x_i))}{\sum_{l=1}^M (\prod_{i=1}^n \mu_{A_i^l}(x_i))}, \quad (11)$$

where $x \in U \subset R^n$ is the input to the fuzzy system, and $f(x) \in V \subset R$ is the output of the fuzzy system.

4. Tracking control design procedure

In this section, a design approach for robust control of uncertain systems is proposed, based on Lyapunov Stability Theory. This approach extends the works of Slotine [23] and later Leitmann and Corless [6], to develop a control structure with great flexibility, specially suitable for Fuzzy Logic applications. It defines a *general class* of manifolds that can be used to achieve convergence of the system trajectory to the neighborhood of the reference trajectory, while leaving considerable freedom for shaping the behavior of the system's trajectory inside this neighborhood.

Control system designers have great freedom in the choice of functions s_e and $\phi(e)$ presented in the following Proposition, allowing specific tailoring for the application at hand. In spite of the inherent flexibility, the closed-loop system is *guaranteed to be robustly stable and has a guaranteed rate of convergence to an error manifold*.

4.1. Preliminary results

Definition 1 (Uniform exponential convergence to a manifold). The system $\dot{x} = f[t, x(t)]$ is said to be uniformly exponentially convergent to within a radius r_B of the manifold $B(x) = 0$ iff there exists $\alpha_B > 0$ with the property that, for any initial conditions $t_0 \in R$ and $x_0 \in D \subset R^n$, there exists $c_B(x_0) \geq 0$ such that, if $x(\cdot)$ is a solution of $\dot{x} = f[t, x(t)]$ with $x(t_0) = x_0$, then:

$$\|B(x(t))\| \leq r_B + c_B(x_0) \exp[-\alpha_B(t - t_0)] \tag{12}$$

for all $t \geq t_0$. Moreover, if $D = R^n$ then the system is globally uniformly exponentially convergent to within a radius r_B of the manifold $B(x) = 0$.

Definition 2 (Class \mathcal{K} (\mathcal{KL}) [13]). A continuous function $\alpha: [0, a) \rightarrow [0, \infty)$ is said to belong to class \mathcal{K} if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class \mathcal{K}_∞ if $a = \infty$ and $\alpha(r) \rightarrow \infty$ as $r \rightarrow \infty$.

A continuous function $\beta: [0, a) \times [0, \infty) \rightarrow [0, \infty)$ is said to belong to class \mathcal{KL} if, for each fixed s , the mapping $\beta(r, s)$ belongs to class \mathcal{K} with respect to r and, for each fixed r , the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \rightarrow 0$ as $s \rightarrow \infty$.

Theorem 3 (Corless [5]). Suppose that for a system $\dot{x} = f[t, x(t)]$ $x \in R^n$, $t \in R_+$ and a continuously differentiable function $B(x): R^n \rightarrow R^n$, there exists a continuously differentiable function $V(x): R^n \rightarrow R$ with the following properties:

(i) There are scalars $\omega_1, \omega_2 > 0$ such that

$$\omega_1 \|B(x)\|^2 \leq V(x) \leq \omega_2 \|B(x)\|^2 \quad \forall x \in R^n. \tag{13}$$

(ii) There are scalars $V^* \geq 0$, $\alpha_B \geq 0$ such that

$$\forall t \in R: \quad \frac{dV(x)}{dt} \leq -2\alpha_B [V(x) - V^*] \forall x \quad \text{s.t. } V(x) > V^*. \tag{14}$$

Then the system is uniformly exponentially convergent to within a radius r_B of the manifold $B(x) = 0$ with rate α_B , where

$$r_B = \sqrt{\frac{V^*}{\omega_1}}, \tag{15}$$

$$c_B(x_0) \triangleq \begin{cases} 0 & \text{if } V(x_0) \leq V^*, \\ \sqrt{\frac{V(x_0) - V^*}{\omega_1}} & \text{if } V(x_0) > V^*. \end{cases} \tag{16}$$

4.2. Tracking control of mechanical systems

Consider a mechanical system described by Eq. (1) having characteristics given by Eqs. (2)–(8). The objective is to design a control Q such that the state $z = [q \ \dot{q}]^T$ of the system follows the state of

a model system $z_m = [q_m \ \dot{q}_m]^T$ in the presence of parameter uncertainties and bounded disturbances. It is assumed $q_m(t), \dot{q}_m(t) \in R^n$ satisfy the equation

$$M_m(q_m)\ddot{q}_m + \bar{C}_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) = v_c \quad (17)$$

provided that

$$\|q_m\| \leq w_{m1} \quad \text{and} \quad \|\dot{q}_m\| \leq w_{m2}. \quad (18)$$

Proposition 4. Consider the system in Eq. (1) with the following control law:

$$Q = Q_L + Q_r, \quad (19)$$

$$Q_L = -K_d B, \quad (20)$$

$$Q_r = \begin{cases} -s(B) - \alpha^* B / \|B\|_2 & \text{if } \|B\|_2 \geq \varepsilon \\ s_\varepsilon & \text{if } \|B\|_2 < \varepsilon \end{cases}, \quad \varepsilon \in R_+, \quad \varepsilon \leq r_B \quad (21)$$

with definitions

$$e \equiv q - q_m, \quad (22)$$

$$B \equiv \dot{e} + \Lambda \phi(e), \quad (23)$$

$$\alpha^* \equiv \lambda_M \left\| \ddot{q}_m - \Lambda \frac{\partial \phi}{\partial e} \dot{e} \right\|_2 + v_b \|\dot{q}\|_2^2 + g_0 + \Theta + \eta \lambda_M \|B\|_2, \quad (24)$$

$$s(B) \equiv [s_1(B) \ \cdots \ s_n(B)]^T, \quad (25)$$

$$\phi(e) \equiv [\phi_1(e_1) \ \phi_2(e_2) \ \cdots \ \phi_n(e_n)]^T, \quad (26)$$

where s_ε is an arbitrary function.

Function $\phi(e)$ is a diffeomorphism, K_d and Λ are constant diagonal positive-definite matrices, $\eta > 0$ is a constant, and the following conditions are satisfied:

$$B_i s_i(B) \geq 0, \quad i = 1, \dots, n, \quad (27)$$

$$\phi_i(0) = 0, \quad \frac{d\phi_i}{de_i} > 0, \quad i = 1, \dots, n, \quad (28)$$

$$\exists k_\phi(\cdot), K_\phi(\cdot) \text{ class } \mathcal{K} \text{ functions: } k_\phi(\|e\|_2) \leq \|\phi(e)\|_2^2 \leq K_\phi(\|e\|_2). \quad (29)$$

Assume that the initial conditions satisfy

$$\begin{aligned} \|q(0)\|_2 &\leq w_1, \\ \|\dot{q}(0)\|_2 &\leq r_v^+ + w_{m2} \leq \frac{\lambda_{\min}(K_d)}{v_b}, \end{aligned} \quad (30)$$

where $w_1 > 0$ is constant, $\lambda_{\min}(K_d)$ is the minimum eigenvalue of K_d and r_v^+ is a positive constant that depends on design parameters and initial conditions. Assume also that condition (18) is satisfied.

Then the closed-loop system has the following properties:

- (i) The system converges exponentially to a manifold: $\|B(e(t), \dot{e}(t))\|_2 \leq r_B + \gamma_B \exp(-\eta t)$, where γ_B and r_B are positive constants that depend on design parameters and initial conditions.
- (ii) The tracking error is input-to-state stable: $\|e(t)\|_2 \leq \beta(\|e(0)\|_2, t) + \gamma(\sup_{0 \leq \tau \leq t} \|B(\tau)\|_2)$, where β is a class \mathcal{KL} function and γ is a class \mathcal{K} function.

Proof. Choose the following function:

$$E = \frac{1}{2} B^T M B. \tag{31}$$

Differentiation with respect to time, substitution of Eqs. (1), (19) and (20) in definition (31) results in

$$\dot{E} = B^T(-\bar{C}\dot{q} - g - K_d B + Q_r + \tau_d) - B^T M \ddot{q}_m + B^T M A \frac{\partial \phi}{\partial e} \dot{e} + \frac{1}{2} B^T \dot{M} B. \tag{32}$$

Using Eq. (5) it is obtained $\frac{1}{2} B^T \dot{M}(q) B = B^T \bar{C}(q, \dot{q}) B$, and denoting $f = M \ddot{q}_m + \bar{C}\dot{q} + g$ yields

$$\dot{E} = B^T \left(Q_r - f + M A \frac{\partial \phi}{\partial e} \dot{e} + \tau_d \right) - B^T (K_d - \bar{C}) B. \tag{33}$$

In order to establish positivity of the last term in the above, we need the matrix $(K_d - \bar{C})$ to be positive definite, then:

$$\dot{q}^T (K_d - \bar{C}) \dot{q} > 0 \Rightarrow \dot{q}^T K_d \dot{q} > \dot{q}^T \bar{C} \dot{q}. \tag{34}$$

From matrix properties we know that $\lambda_{\min}(K_d) \|\dot{q}\|_2^2 \leq \dot{q}^T K_d \dot{q} \leq \lambda_{\max}(K_d) \|\dot{q}\|_2^2$. Hence, using inequality (3) it is obtained that $B^T (K_d - \bar{C}) B > 0$ if:

$$\|\dot{q}(t)\|_2 < \frac{\lambda_{\min}(K_d)}{v_b}. \tag{35}$$

The above inequality is satisfied provided that the initial conditions satisfy condition (30). Using Eq. (33) we get

$$\dot{E} \leq B^T \left(Q_r - f + M A \frac{\partial \phi}{\partial e} \dot{e} + \tau_d \right). \tag{36}$$

Property (i). Define a state $x \equiv [e \ \dot{e}]^T$ and apply inequality (2) to Eq. (31):

$$\frac{1}{2} \lambda_m \|B\|_2^2 \leq \frac{1}{2} B^T M B \leq \frac{1}{2} \lambda_M \|B\|_2^2. \tag{37}$$

Condition (13) is satisfied for $V \equiv E$, with $\omega_1 \equiv \frac{1}{2} \lambda_m$ and $\omega_2 \equiv \frac{1}{2} \lambda_M$. To satisfy condition (14), we need

$$\dot{E} \leq -2\eta(E - E^+), \tag{38}$$

where $V^* \equiv E^+ > 0$ is a design parameter. From inequalities (36) and (38), we get

$$B^T \left(Q_r - f + M A \frac{\partial \phi}{\partial e} \dot{e} + \tau_d \right) \leq -2\eta(E - E^+). \tag{39}$$

Consider the control Q given in (19)–(21) with

$$Q_r = -s(B) - \alpha^* \frac{B}{\|B\|_2}. \quad (40)$$

Rearranging inequality (39) yields

$$\begin{aligned} & \left[\lambda_M \|\ddot{q}_m - A \frac{\partial \phi}{\partial e} \dot{e}\|_2 + v_b \|\dot{q}\|_2^2 + g_0 + \Theta + \eta \lambda_M \|B\|_2 \right] \|B\|_2 + B^T s(B) \\ & \geq -B^T \left[M \left(\ddot{q}_m - A \frac{\partial \phi}{\partial e} \dot{e} \right) + \bar{C} \dot{q} + g - \tau_d \right] + \eta B^T M B - 2\eta E^+. \end{aligned} \quad (41)$$

From properties (2), (3) and (6) it is obtained:

$$\begin{aligned} \|B\|_2 \lambda_M \left\| \ddot{q}_m - A \frac{\partial \phi}{\partial e} \dot{e} \right\|_2 & \geq -B^T M \left(\ddot{q}_m - A \frac{\partial \phi}{\partial e} \dot{e} \right), \\ \|B\|_2 v_b \|\dot{q}\|_2^2 & \geq -B^T \bar{C} \dot{q}, \\ (g_0 + \Theta) \|B\|_2 & \geq -B^T (g - \tau_d), \\ \eta \lambda_M \|B\|_2^2 & \geq \eta B^T M B. \end{aligned} \quad (42)$$

Hence, using condition (27) we have exponential convergence to the manifold $B=0$, according to Theorem 3, with

$$r_B = \sqrt{\frac{E^+}{\omega_1}} = \sqrt{\frac{2E^+}{\lambda_m}}, \quad (43)$$

$$\gamma_B = \begin{cases} 0 & \text{if } \frac{1}{2} \lambda_M \|B_0\|_2^2 \leq E^+ \\ \sqrt{\frac{(1/2) \lambda_M \|B_0\|_2^2 - E^+}{(1/2) \lambda_m}} = \sqrt{\frac{\lambda_M \|B_0\|_2^2}{\lambda_m} - r_B^2} & \text{if } \frac{1}{2} \lambda_M \|B_0\|_2^2 > E^+ \end{cases} \quad (44)$$

and parameter ε satisfying

$$\varepsilon \leq r_B \Rightarrow \varepsilon \leq \sqrt{\frac{2E^+}{\lambda_m}}. \quad (45)$$

Property (ii). Consider the system

$$\dot{e} + A\phi(e) = u(e, \dot{e}) \equiv B \Rightarrow \dot{e} = B - A\phi(e). \quad (46)$$

Choose the function

$$\mathcal{V} = \frac{1}{2} \phi^T(e) \phi(e). \quad (47)$$

Then, condition (A.3) is satisfied by using condition (29). Differentiating Eq. (47) and applying Eq. (46) and definition (26) yields

$$\dot{\mathcal{V}} = \sum_{i=1}^{i=n} \left[-\Lambda_{ii} \phi_i^2 \frac{d\phi_i}{de_i} + \phi_i \frac{d\phi_i}{de_i} B_i \right]. \tag{48}$$

Using inequality (28) on condition (A.4) yields

$$\Lambda_{ii} \phi_i^2 \frac{d\phi_i}{de_i} \geq \phi_i \frac{d\phi_i}{de_i} B_i \Rightarrow \Lambda_{ii} \phi_i^2 \geq \phi_i B_i \quad \text{for } \|e\|_2 \geq \rho(\|B\|_2) > 0. \tag{49}$$

By requiring

$$\Lambda_{ii} |\phi_i|^2 = \Lambda_{ii} \phi_i^2 \geq |\phi_i| |B_i| \geq \phi_i B_i, \quad |\phi_i| \geq \frac{\|B\|_2}{\Lambda_{ii}} \geq \frac{|B_i|}{\Lambda_{ii}}, \tag{50}$$

we get

$$|\phi_i(e_i)| \geq \frac{\|B\|_2}{\Lambda_{ii}} \Rightarrow e_i \geq \phi_i^{-1} \left(\frac{\|B\|_2}{\Lambda_{ii}} \right) \Rightarrow e_i^2 \geq \left[\phi_i^{-1} \left(\frac{\|B\|_2}{\Lambda_{ii}} \right) \right]^2 \tag{51}$$

and

$$\|e\|_2 \geq \sqrt{\sum_{i=1}^{i=n} \left[\phi_i^{-1} \left(\frac{\|B\|_2}{\Lambda_{ii}} \right) \right]^2} \equiv \rho(\|B\|_2), \tag{52}$$

satisfying condition (A.4) and proving Property (ii). \square

For the *special case* of a linear manifold B , we can prove exponential convergence of the tracking error as follows.

Corollary 5. Consider the controller given by Eqs. (19)–(26) and satisfying conditions (27)–(30). For

$$B \equiv \dot{e} + \Lambda e \tag{53}$$

and

$$\eta > \lambda_{\max}(\Lambda) - \lambda_{\min}(\Lambda), \tag{54}$$

where $\lambda_{\min}(\Lambda)$ and $\lambda_{\max}(\Lambda)$ are the minimum and maximum eigenvalues of Λ , we have

- (c-i) $\|e(t)\|_2 \leq r_e + \gamma_e \exp(-\eta_e t)$,
- (c-ii) $\|\dot{e}(t)\|_2 \leq r_v + \gamma_v \exp(-\eta_v t)$,

where γ_e , r_e , η_e , η_v , r_v , and γ_v are positive constants that depend on design parameters and initial conditions.

For proof, see [14].

Notes:

- (i) The radius of convergence r_B and ε are *design parameters*.
- (ii) Possible choices for $s(B)$ in definition (21) are

$$s(B) = [0] \in R^n, \quad s_i(B_i) = K_i^* B_i. \tag{55}$$

- (iii) For $\|B(x)\|_2 < \varepsilon$ the control is *arbitrary*. A possible choice is

$$Q_r = s_\varepsilon \equiv -s(B) - \alpha_\varepsilon^* \frac{B}{\varepsilon} \quad \text{for } \|B(x)\|_2 < \varepsilon, \tag{56}$$

thus satisfying continuity at $\|B(x)\|_2 = \varepsilon$. Moreover, for $\|B(x)\|_2 < \varepsilon$ the control law constitutes a PD control logic with an additional dissipating term.

5. Tracking fuzzy logic controllers

In the following section, the performance of the proposed method is demonstrated using two Fuzzy Logic implementations of the control law defined by the proposition in Section 4. The two designs presented here are only *a sample of possible implementations* of the method proposed in this paper. They demonstrate that the method provides the flexible framework needed for fuzzy logic control design. It leaves ample freedom for the implementation of control systems and allows to customize the design to specific applications.

All control system implementations use the structure shown in Fig. 1. The different implementations are described in the following.

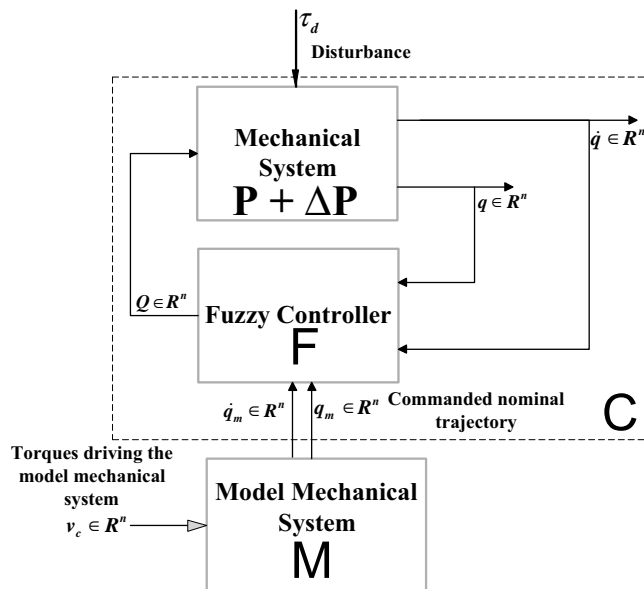


Fig. 1. Closed-loop fuzzy tracking control system.

5.1. Fuzzy manifold-boundary robust controller

In the fuzzy manifold-boundary robust controller implementation, Q_L is a conventional PD control law and Q_r has a fuzzy component, see Eqs. (20) and (21).

From definition (21), we get that the control for $\|B\| < \varepsilon$ is given by an arbitrary function s_ε . However, the choice of s_ε affects the transient response of the system. In particular, if $s_\varepsilon = 0$ then high-frequency chattering may occur. The controller in this section eliminates chattering by implementing a fuzzy s_ε .

5.1.1. Design of Q_r

For $\|B\|_2 > \varepsilon$, Q_r is implemented using Eq. (21) with $s(B) = 0$. For $\|B\|_2 \leq \varepsilon$, Q_r is given by

$$Q_r = s_\varepsilon = -\alpha^* K_f \frac{B}{\varepsilon}, \tag{57}$$

where K_f is the crisp output of a Fuzzy Logic Controller described as follows.

Step 1: Choice of membership functions. Define

$$\sigma_\varepsilon = \frac{\|B\|_2}{\varepsilon}. \tag{58}$$

Choose N fuzzy membership functions for σ_ε . The centers of the membership functions satisfy:

$$\frac{\sigma_\varepsilon^l}{\varepsilon} \geq 0, \quad \frac{\sigma_\varepsilon^l}{\varepsilon} \leq 1, \quad \frac{\sigma_\varepsilon^l}{\varepsilon} < \frac{\sigma_\varepsilon^{l+1}}{\varepsilon}. \tag{59}$$

Step 2: Formulation of fuzzy rules. Formulate the IF–THEN rules of the form

$$\text{IF } \sigma_\varepsilon \text{ is } A_{\sigma_\varepsilon}^l \text{ THEN } K_f \text{ is } B_{K_f}^l, \tag{60}$$

where $l = 1, 2, \dots, N$. The centers of the THEN parts of the fuzzy sets $B_{K_f}^l$, denoted by $c_{K_f}^l$, are chosen so that:

$$0 \leq \sigma_\varepsilon c_{K_f}^l \leq 1; \quad c_{K_f}^l < c_{K_f}^{l+1} \tag{61}$$

Step 3: Obtaining crisp values. The output is given by

$$K_f = \frac{\sum_{l=1}^{l=N} \mu_{A^l}(\sigma_\varepsilon) c_{K_f}^l}{\sum_{l=1}^{l=N} \mu_{A^l}(\sigma_\varepsilon)}. \tag{62}$$

This FLC allows precise shaping of the gain function for $\|B\|_2 \leq \varepsilon$ in order to achieve better tracking errors. Fig. 2 shows possible shapes of fuzzy gain functions $K_f \|B\|/\varepsilon$, chosen arbitrarily. Note that the corresponding graph for the new non-fuzzy robust controller, as given by Eq. (56), is a straight line from lower left corner $(\sigma_\varepsilon, K_f \sigma_\varepsilon) = (0, 0)$ to upper right corner $(\sigma_\varepsilon, K_f \sigma_\varepsilon) = (1, 1)$.

5.2. Fuzzy manifold robust controller

In the fuzzy manifold robust controller, the error manifold $B = 0$ is implemented using fuzzy logic, and Q_r has the fuzzy component described for the fuzzy manifold boundary robust controller.

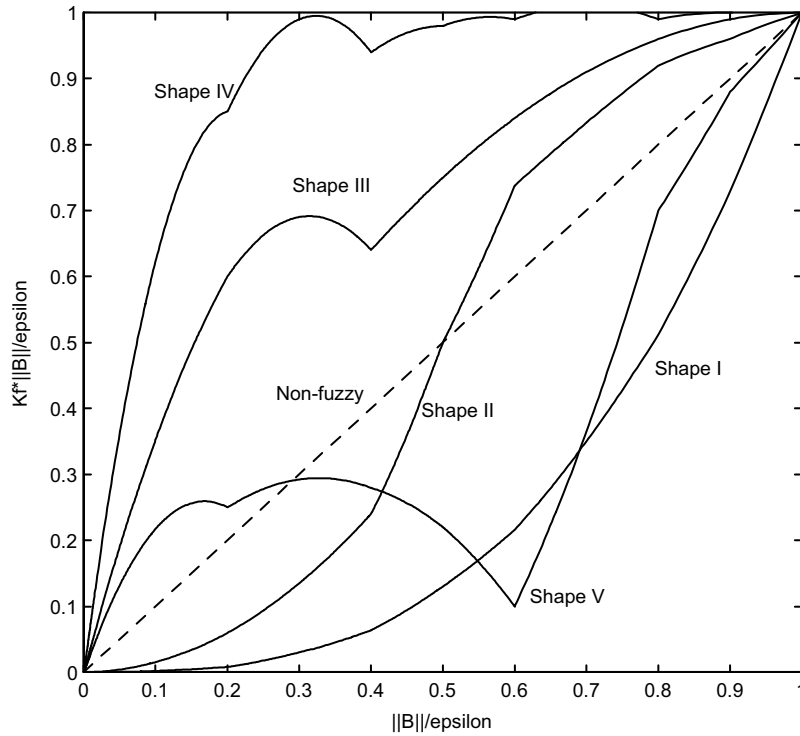


Fig. 2. Fuzzy gain functions: shapes I–V and linear gain (non-fuzzy).

To choose the function $\phi(e)$ for our controller, we assume that on the error manifold $B = \dot{e} + \Lambda\phi(e) = 0$ the system behaves as a set of n decoupled subsystems. We therefore choose a manifold that minimizes a performance criteria for the a system of unit masses in the absence of friction:

$$\ddot{e}_i = u_i, \quad |u_i| \leq 1. \tag{63}$$

In terms of the state-space variables

$$e_{i1} \equiv e_i, \quad e_{i2} \equiv \dot{e}_i, \quad \mathbf{x}_i = [e_{i1} \quad e_{i2}]^T, \tag{64}$$

the equations of motion are

$$\dot{x}_{i1} = x_{i2}, \quad \dot{x}_{i2} = u_i, \quad |u_i| \leq 1. \tag{65}$$

In particular, in order to minimize the time to reach the origin

$$e_i = \dot{e}_i = 0, \quad i = 1, \dots, n \tag{66}$$

the solution is (see [32])

$$u_i^*(\mathbf{x}) = \begin{cases} -\text{sgn}[\sigma_i(\mathbf{x}_i)] & \text{if } \sigma_i(\mathbf{x}_i) \neq 0, \\ -\text{sgn}[x_{i2}] & \text{if } \sigma_i(\mathbf{x}_i) = 0 \text{ and } \mathbf{x}_i \neq 0, \\ 0 & \text{if } \mathbf{x}_i = 0, \end{cases} \tag{67}$$

where the switching surface (error manifold) $\sigma_i(\mathbf{x}) = 0$ is

$$\sigma_i(\mathbf{x}_i) = \begin{cases} x_{i2} + \sqrt{2x_{i1}} & \text{if } x_{i1} \geq 0, \\ x_{i2} - \sqrt{2|x_{i1}|} & \text{if } x_{i1} \leq 0. \end{cases} \quad (68)$$

Based on the discussion above, the error manifold chosen is

$$\begin{aligned} B &= \dot{e} + \Lambda\phi(e), \quad \phi(e) = \text{diag}[\phi_1(e_1) \dots \phi_n(e_n)]^T, \\ \phi_i(e_i) &= \text{sign}(e_i)\sqrt{2|e_i|}. \end{aligned} \quad (69)$$

Note that condition (28) is satisfied

$$\phi'_i(e_i) \equiv \frac{d\phi_i}{de_i} = \frac{1}{2\sqrt{|e_i|}} > 0. \quad (70)$$

It can be seen that $\phi'_i(e_i)$ is discontinuous at $e_i = 0$. To overcome this problem we approximate $\phi(e)$ given in Eq. (69) by a continuously differentiable fuzzy logic function. To improve performance further, we use the fuzzy s_ε described previously for $\|B\| \leq \varepsilon$ (see Eq. (57)). The resulting controller, denoted by fuzzy manifold robust controller, is described in the following.

Step 1: Choice of membership functions. Choose N fuzzy membership functions for each e_i . Their centers satisfy

$$\underline{e}_i^l < \underline{e}_i^{l+1}, \quad (71)$$

where $l = 1, 2, \dots, N$ and $i = 1, 2$.

Step 2: Formulation of fuzzy rules. Formulate the IF–THEN rules of the form

$$\text{IF } e_i \text{ is } A_{e_i}^l \text{ THEN } \phi \text{ is } B_{\phi_i}^l. \quad (72)$$

The centers of the THEN parts of the fuzzy sets $B_{\phi_i}^l$, denoted by $c_{\phi_i}^l$, satisfy

$$c_{\phi_i}^l = \text{sign}(\underline{e}_i^l)\sqrt{|\underline{e}_i^l|}, \quad (73)$$

where \underline{e}_i^l is the center of membership function $A_{e_i}^l$.

Step 3: Obtaining crisp values. The output is given by

$$\phi_i = \frac{\sum_{l=1}^N \mu_{A^l}(e_i)c_{\phi_i}^l}{\sum_{l=1}^N \mu_{A^l}(e_i)} \quad (74)$$

which is continuously differentiable if $\mu_{A^l}(e_i)$ is continuously differentiable.

6. Simulation

Consider a two-link planar robotic arm shown in Fig. 3. The vector of generalized coordinates is $q = [x \ \theta]^T$ where x is the second joint displacement and θ is the first joint angle. The corresponding generalized forces vector is $Q = [F \ \tau]^T$.

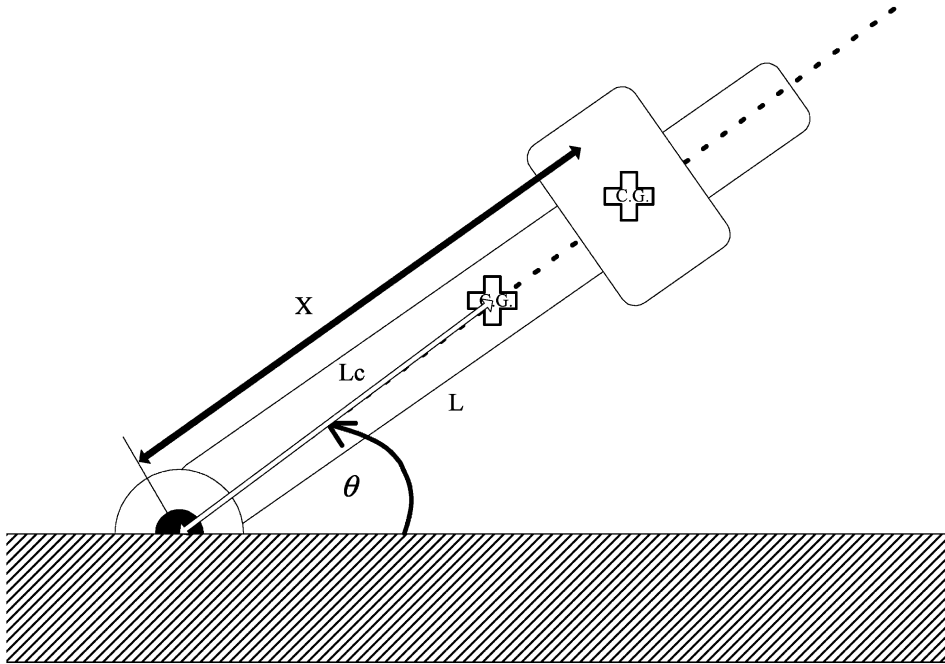


Fig. 3. Two-degree-of-freedom system.

Assuming uniform rigid links, it can be shown that the equation of motion (1) for the RP arm is given by

$$M(q)\ddot{q} + \bar{C}(q, \dot{q})\dot{q} + g(q) = Q, \quad (75)$$

$$M(q) = \begin{bmatrix} m_1 & 0 \\ 0 & m_1 x^2 + \frac{4}{3} m_2 L_c^2 \end{bmatrix}, \quad (76)$$

$$\bar{C}(q, \dot{q}) = \begin{bmatrix} 0 & -m_1 x \dot{\theta} \\ m_1 x \dot{\theta} & m_1 x \dot{x} \end{bmatrix}, \quad (77)$$

$$g(q) = \begin{bmatrix} m_1 g \cos(\theta) \\ -(m_1 x + m_2 L_c) g \sin(\theta) \end{bmatrix}. \quad (78)$$

The physical properties of the links used for the control systems are given in Table 1. It was assumed that the rotational link is uniform and therefore $L_c = L/2$.

The gain matrices were

$$A = \begin{bmatrix} 10.0 & 0 \\ 0 & 10.0 \end{bmatrix}, \quad K_d = \begin{bmatrix} 40.0 & 0 \\ 0 & 40.0 \end{bmatrix}.$$

Table 1
Parameters of the RP robot arm

	True	Minimum	Nominal	Maximum
Mass of prismatic link (kg)	5.0	4.0	4.8	10.0
Mass of rotational link (kg)	2.0	1.0	2.9	3.0
Length of rotational link (m)	1.0	0.99	1.0	1.1

In all simulation cases, the system was required to follow a model system given by the dynamics equation (1) with the following parameters:

$$M_m(q) = \begin{bmatrix} m_{1_m} a_{1_m}^2 & 0 \\ 0 & m_{2_m} a_{2_m}^2 \end{bmatrix}, \quad C_m(q, \dot{q}) = \begin{bmatrix} 9.72 \dot{\theta}_1 \\ 7.90 \dot{\theta}_2 \end{bmatrix}, \quad g_m(q) = \begin{bmatrix} m_{1_m} g a_{1_m} \theta_1 \\ m_{2_m} g a_{2_m} \theta_2 \end{bmatrix},$$

$$m_{1_m} = 1.0, \quad m_{2_m} = 1.5, \quad a_{1_m} = 3.0, \quad a_{2_m} = 2.0, \quad g = 9.81.$$

The model's reference input, Q_m , was

$$Q_{1_m} = Q_{2_m} = \begin{cases} t & \text{for } t \leq 8(\text{s}), \\ 4(1.0 + \cos[\frac{2\pi}{5}(t - 8)]) & \text{for } t > 8(\text{s}) \end{cases} \quad (79)$$

with the initial conditions

$$q_{m_0} = [0.35 \quad 0.0]^T, \quad \dot{q}_{m_0} = [0.0 \quad 0.0]^T.$$

The initial conditions for the plant were assumed to be

$$q_0 = [0.5 \quad 0.0]^T, \quad \dot{q}_0 = [0.0 \quad 0.0]^T.$$

Resulting bounds on model rates and accelerations were

$$\sup(\|\ddot{q}_m\|_2) = 0.4483, \quad \sup(\|\dot{q}_m\|_2) = 0.3465. \quad (80)$$

The output path, rates and accelerations of the model are shown in Fig. 4. In all the simulations, the following design parameters were chosen (see Eqs. (21) and (24)):

$$\varepsilon = 5.9 \times 10^{-6}, \quad \eta = 100. \quad (81)$$

6.1. Robotic systems control laws

The following control laws were compared with the proposed control law via MATLAB simulations. For detailed discussion of these methods, see [1, 17].

6.1.1. Inverse dynamics control law

For a mechanical system given by Eq. (1), the inverse dynamics control law is given by

$$Q = M(q)u_0 + C(q, \dot{q}) + g(q), \quad (82)$$

$$u_0 = \ddot{q}_m + K_d(\dot{q}_m - \dot{q}) + K_p(q_m - q), \quad (83)$$

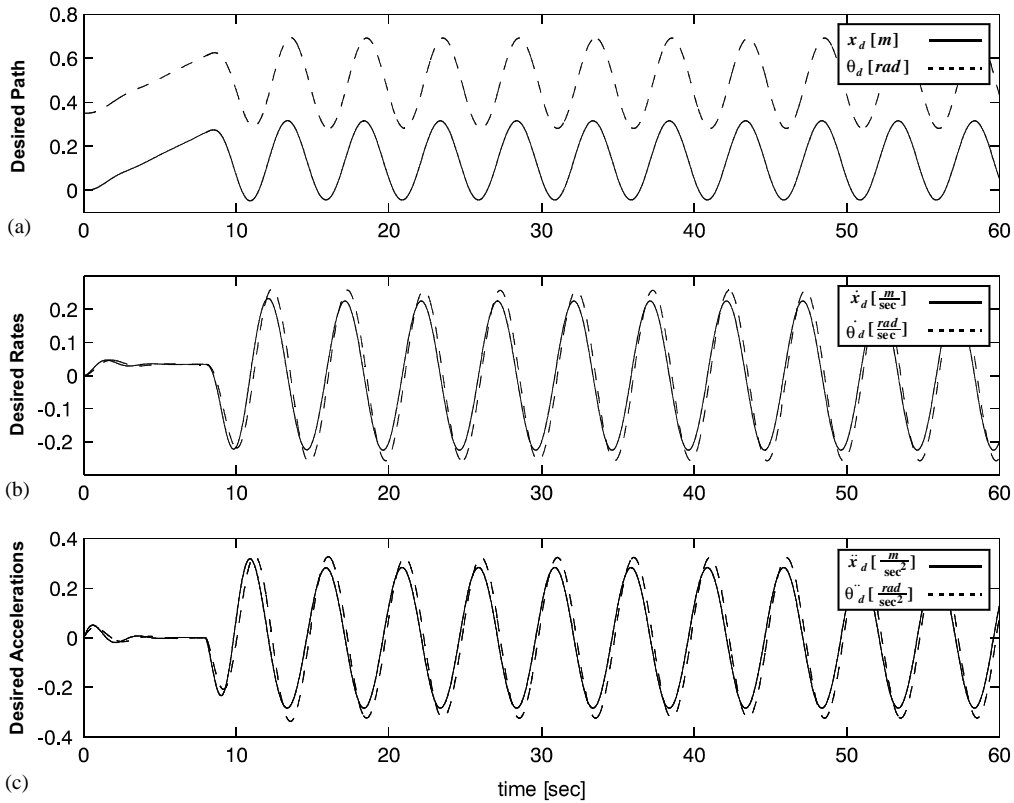


Fig. 4. (a) Desired path, (b) desired rates, (c) desired accelerations.

where $e = q - q_m$. The gain matrices taken are diagonal positive-definite:

$$K_d = \text{diag}(k_{d_1}, k_{d_2}), \quad K_p = \text{diag}(k_{p_1}, k_{p_2}) \quad (84)$$

with $k_{p_1} = k_{p_2} = 400$ and $k_{d_1} = k_{d_2} = 40$. These are the same gains used for the proposed control law.

6.1.2. Robust saturation-type control law

The robust saturation-type control law is given by

$$Q = Q_N + Q_L + Q_\eta. \quad (85)$$

The inner non-linear control is given by

$$Q_N = \hat{C}(q, \dot{q}) + \hat{g}(q), \quad (86)$$

$$Q_\eta = \hat{M}(q)v. \quad (87)$$

Define the following constants and functions:

$$\alpha_r = \frac{\lambda_M - \lambda_m}{\lambda_m + \lambda_M}, \quad K = [K_p \quad K_d], \quad \Phi = v_b \|\dot{q}\|^2 + g_0,$$

$$\rho(e, t) = \frac{1}{1 - \alpha_r} [\alpha_r Q_1 + \|K\|_{i2} \|e\|_2 + \lambda_M \Phi(e, t)],$$

where λ_M and λ_m are the maximum and minimum eigenvalues of $M(q)$, v_b is defined in (3), g_0 is defined in (6), Q_1 is a finite number bigger than the supremum of the norm of the nominal path accelerations vector, and the 2-induced matrix norm of K is defined by $\|K\|_{i2} = \sqrt{\lambda_{\max}(K^T K)}$. Matrices K_p and K_d are defined by (84) and have the same values.

Then v is chosen to be

$$v = \begin{cases} -\rho(e, t) \frac{\mathbf{B}^T P e}{\|\mathbf{B}^T P e\|_2} & \text{if } \|\mathbf{B}^T P e\|_2 > \varepsilon, \\ -\rho(e, t) \frac{\mathbf{B}^T P e}{\varepsilon} & \text{if } \|\mathbf{B}^T P e\|_2 \leq \varepsilon \end{cases} \quad (88)$$

where $\varepsilon > 0$ is a small constant and P is a solution of Lyapunov's equation:

$$\mathbf{A}^T P + P \mathbf{A} = -\varrho, \quad \varrho = 20.I \quad (89)$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \quad (90)$$

The outer linear control is given by

$$Q_L = \hat{M}(q)\vartheta, \quad (91)$$

$$\vartheta = \ddot{q}_d - K_d(\dot{q} - \dot{q}_m) - K_p(q - q_m), \quad (92)$$

where $\hat{M}(q)$, $\hat{C}(q, \dot{q})$ and $\hat{g}(q)$ denote the nominal values of the mass matrix, Coriolis terms and potential terms, respectively.

6.1.3. Adaptive inertia-related control law

The adaptive control law is given by

$$Q = \hat{M}(q)\dot{v} + \hat{C}(q, \dot{q})v + \hat{g}(q) - K_r B, \quad (93)$$

$$v = \dot{q}_m - \Lambda(q - q_m) = \dot{q}_m - \Lambda e, \quad (94)$$

$$B = \dot{q} - v = \dot{e} + \Lambda e, \quad (95)$$

where Λ and K_r are diagonal positive-definite matrices and $e = q - q_m$. The parameter adaptation law is

$$\frac{d\tilde{Q}}{dt} = \frac{d\hat{Q}}{dt} = -\Gamma^{-1} Y(q, \dot{q}, v, \dot{v})^T B, \quad (96)$$

where Γ is a positive-definite matrix, and $Y(q, \dot{q}, v, \dot{v})$ is the matrix defined by (see property (7)):

$$M(q)\dot{B} + \bar{C}(q, \dot{q})B + K_r B = Y(q, \dot{q}, v, \dot{v})\tilde{q} \tag{97}$$

where $\tilde{q} = \hat{q} - q$.

6.2. Fuzzy manifold-boundary robust controller

The proposed control law was simulated via MATLAB/Simulink for the fuzzy manifold-boundary robust controller (see Section 5.1 and Eqs. (57)–(62)), named “new fuzzy robust controller” throughout this subsection. Its performance was compared to previously described control design methods for robust control, for several shapes of fuzzy gain functions. For the new non-fuzzy robust controller, Q_r for $\|B\| \leq \varepsilon$ is given by Eq. (56) with $s(B) = 0$, so that it could also be implemented using Eq. (57) with $K_f = 1$. Hence, its plot of the “fuzzy gain function” $K_f \|B\| / \varepsilon$ as a function of $\|B\| / \varepsilon$ is linear (see Fig. 2).

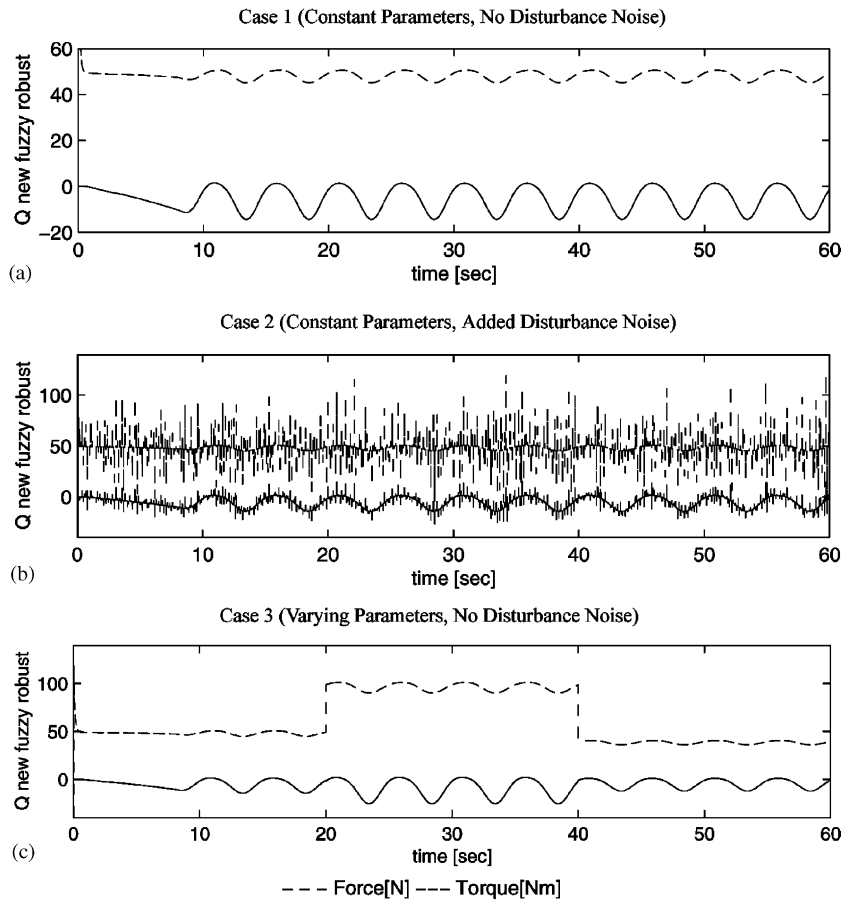


Fig. 5. Comparison of control efforts for the new fuzzy robust controller: (a) Case 1 (constant parameters, no disturbance noise), (b) Case 2 (constant parameters, added disturbance noise), (c) Case 3 (varying parameters, no disturbance noise).

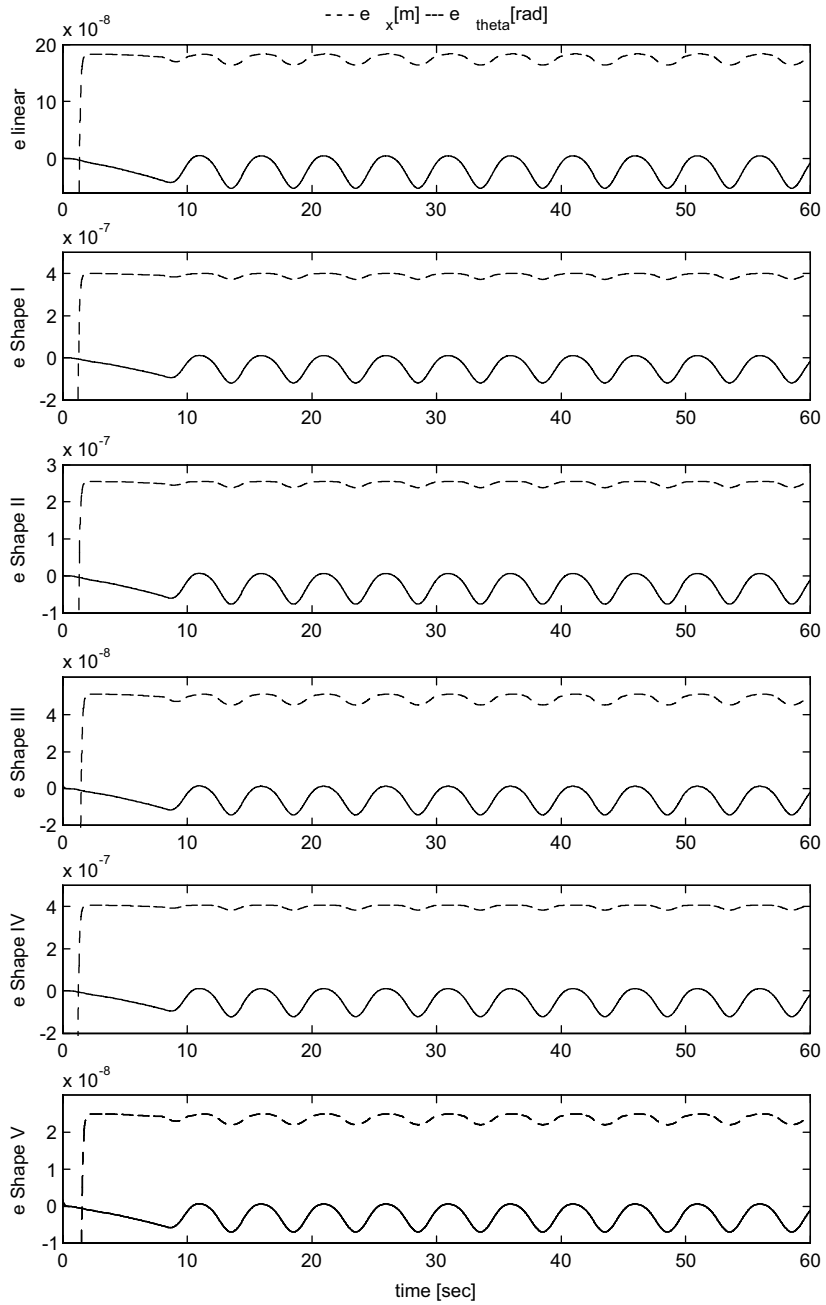


Fig. 6. Comparison of tracking errors for different shapes of fuzzy gain functions, Case 1 (constant parameters, no disturbance noise).

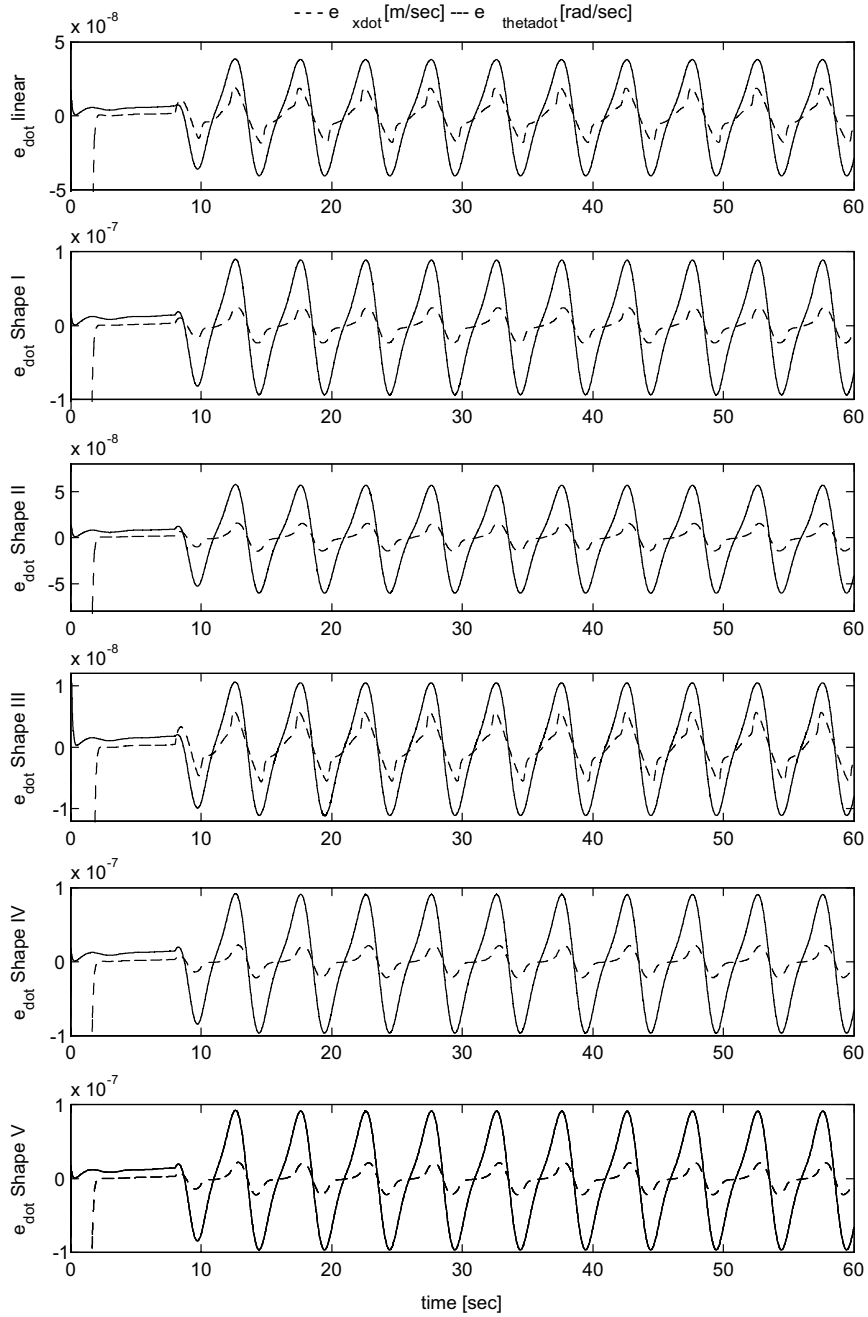


Fig. 7. Comparison of tracking error rates for different shapes of fuzzy gain functions, Case 1 (constant parameters, no disturbance noise).

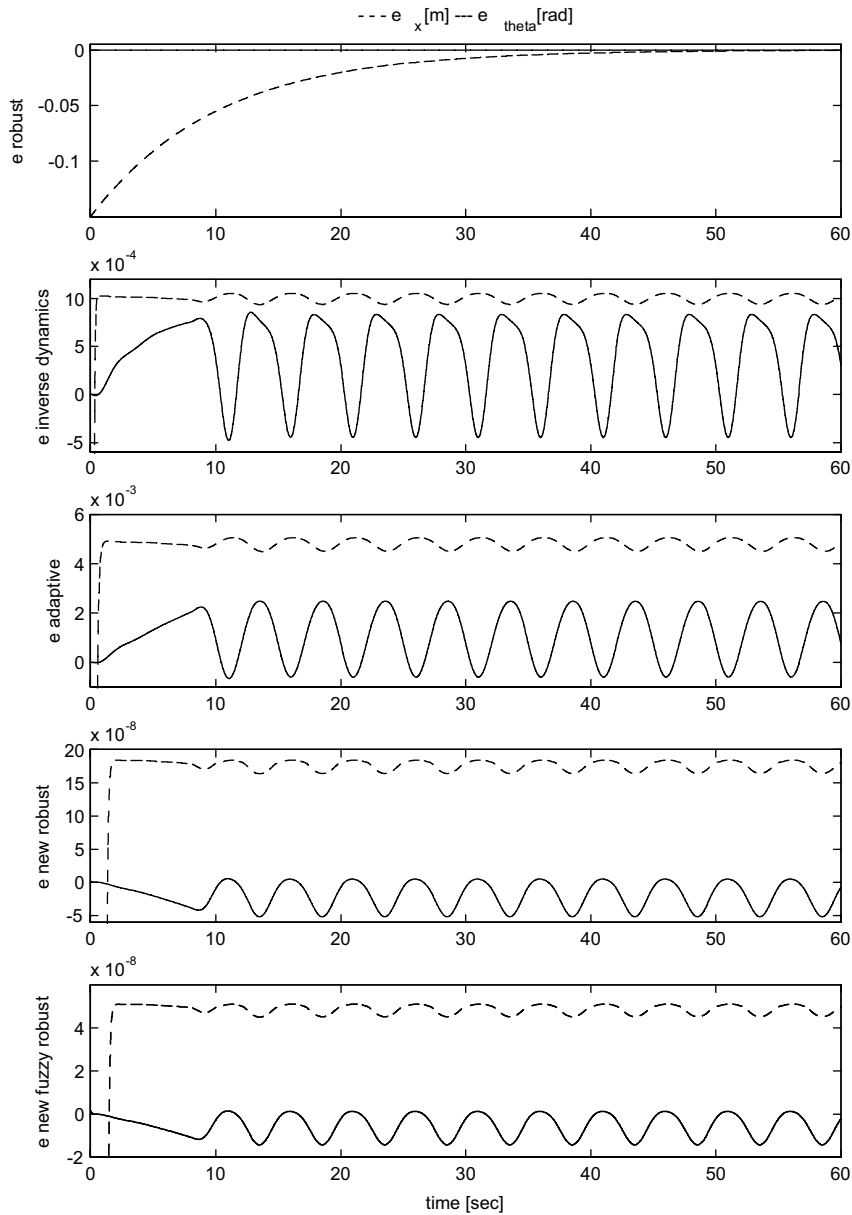


Fig. 8. Comparison of tracking errors, Case 1 (constant parameters, no disturbance noise).

Three simulation cases were evaluated. Fig. 5 shows the control efforts for the new fuzzy robust controller, Case 1 (constant parameters, no disturbance noise), Case 2 (constant parameters, added disturbance noise) and Case 3 (varying parameters, no disturbance noise). It is noted that the behavior of the control efforts for all the evaluated controllers (robust, inverse dynamics, adaptive, new robust and new fuzzy robust) was very similar for each one of the simulation cases, although the magnitudes of the errors were very different, as showed in the following.

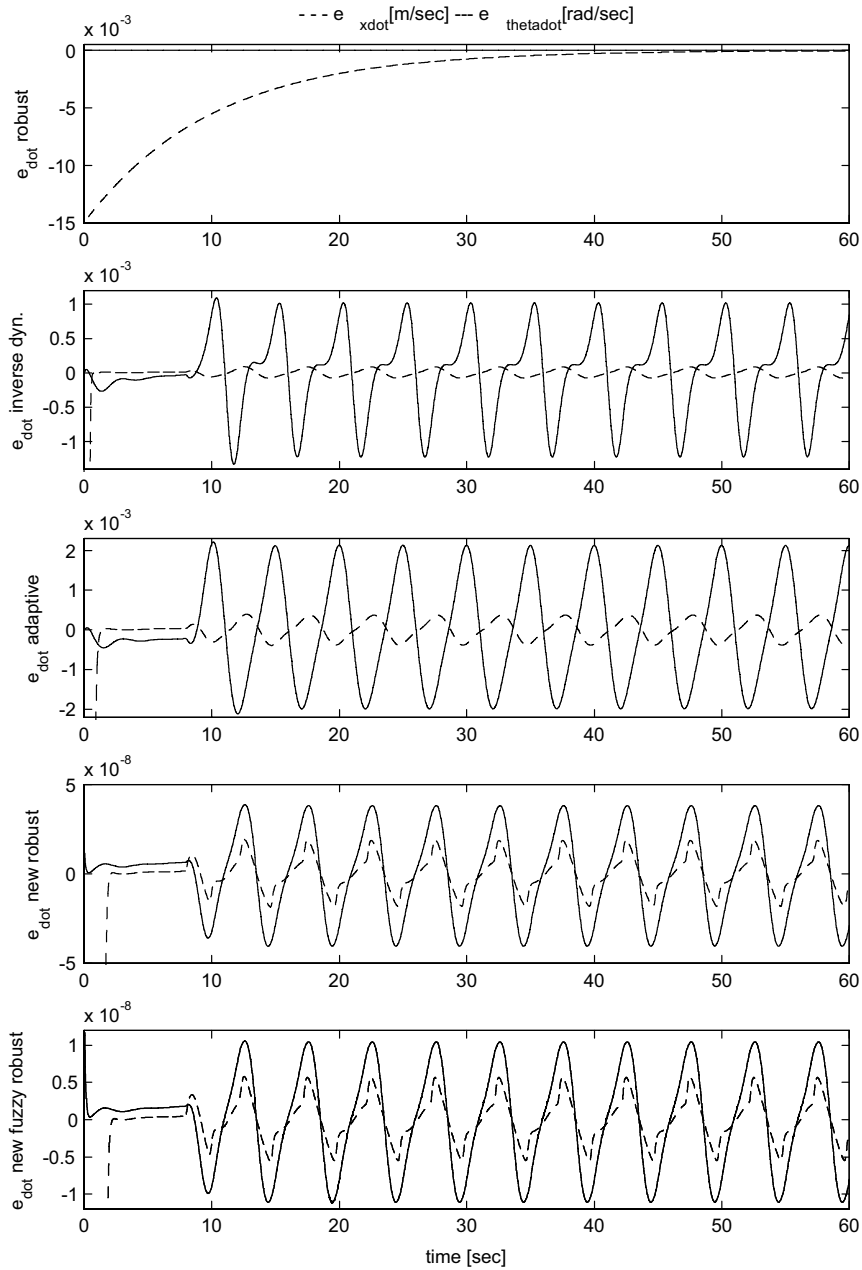


Fig. 9. Comparison of tracking error rates, Case 1 (constant parameters, no disturbance noise).

6.2.1. Case 1: constant parameters, no disturbance noise

Comparison between fuzzy gain functions: The proposed robust controller with linear gain for $\|B\|_2 \leq \varepsilon$, and with five different shapes of fuzzy gain functions shown in Fig. 2 were compared. It was obtained that all shapes (Shapes I–V, and linear) give similar smooth control efforts as shown

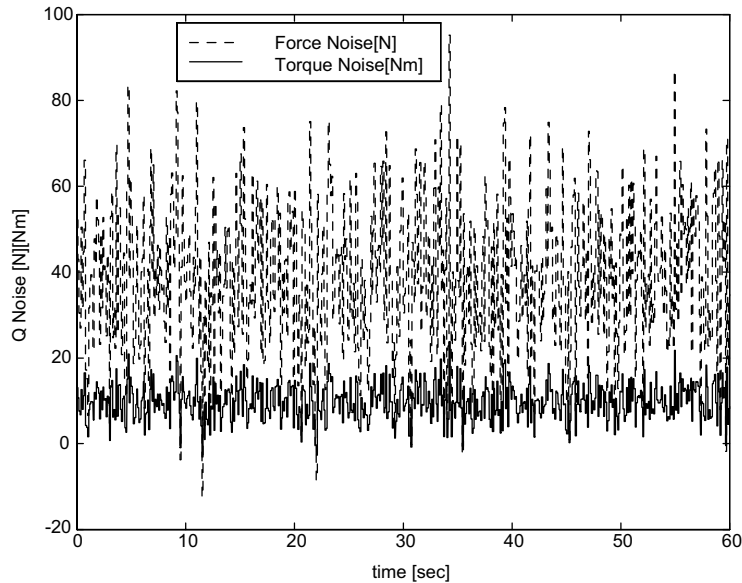


Fig. 10. Disturbance noise added to plant input.

for Case 1 in Fig. 5. The tracking errors and rates are shown in Figs. 6 and 7. The lowest errors are obtained for Shapes III and V which possess local minima. It can be observed that the lowest rate errors are obtained for Shape III.

Comparison between control laws: The comparison between the controllers proposed in this paper and other designs is shown in Figs. 8 and 9. It was obtained that all methods yield similar control efforts as shown for Case 1 in Fig. 5. Fig. 8 shows that the tracking errors for the new non-fuzzy robust controller were *four orders of magnitude* smaller than those obtained for the adaptive and inverse dynamics controllers. It can also be seen that the errors for the new fuzzy robust controller were 25% smaller than those of its non-fuzzy implementation. The tracking rate errors obtained for the new robust controllers were *five orders of magnitude* smaller than those obtained for the robust, inverse and adaptive controllers. The new fuzzy robust controller showed error rates $\frac{1}{5}$ of those obtained for its non-fuzzy implementation as seen in Fig. 9.

6.2.2. Case 2: constant parameters, added disturbance noise

Fig. 10 shows the generated disturbance noise that was added to the output generalized forces of the controllers. The masses of the plant links remained constant throughout the simulation. For the rotational joint, the noise was composed of a magnitude 10 bias and MATLAB's Band Limited White Noise output, for noise power of 2, sample time 0.1 and seed 123456. For the prismatic joint, this noise was amplified by 4. The resulting bound on the noise was $\Theta = 100$ (see inequality (8)).

The comparison between the controllers proposed here and other designs is shown in Figs. 11 and 12. All the time histories of control efforts with added noise were similar to Case 2 in Fig. 5. In Fig. 11, we see that the tracking errors for the controllers proposed in Section 5.1 were not

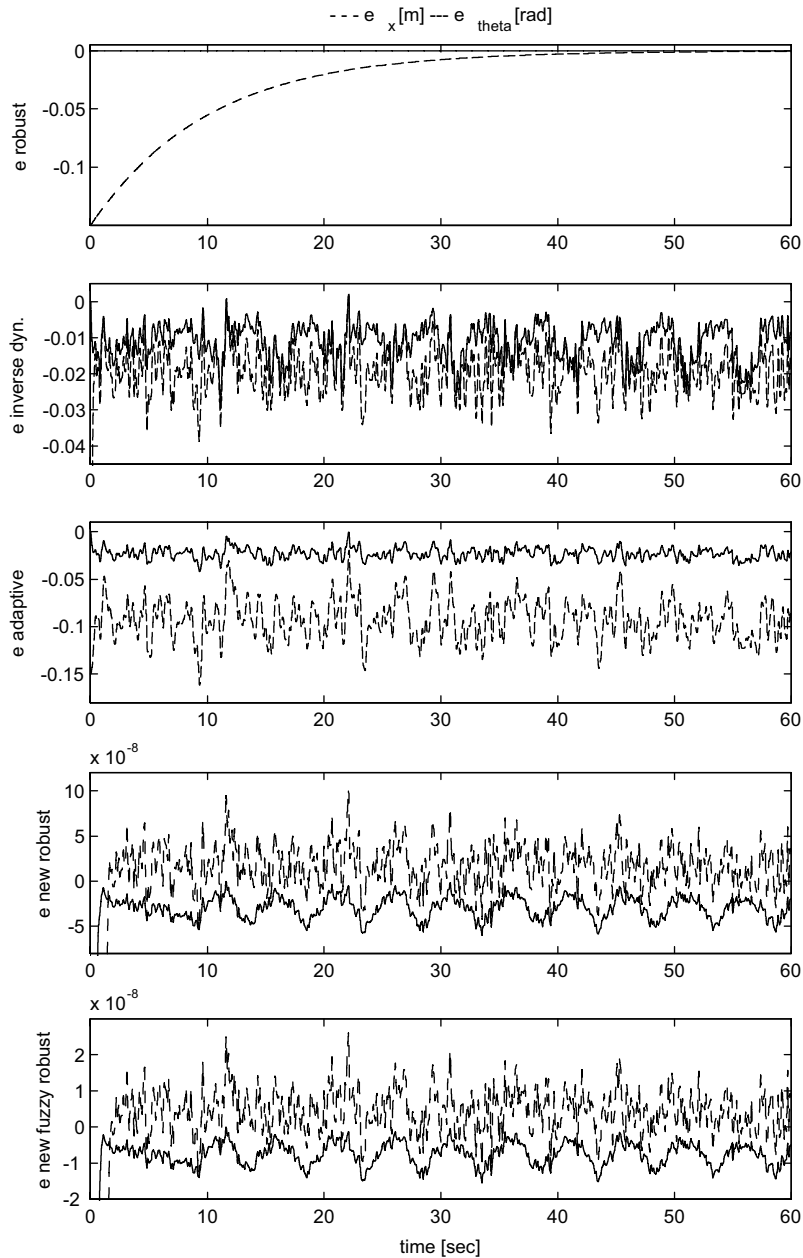


Fig. 11. Comparison of tracking errors, Case 2 (constant parameters, added disturbance noise).

significantly affected by the high value of Θ present in the computation of α^* (see Eq. (21)). Fig. 12 shows that the tracking error rates increased only by a *factor of 4* for the new fuzzy robust controller and a *factor of 10* for the new non-fuzzy robust controller. For the other control designs given in Section 6.1 the rate errors increased by *two orders of magnitude*.

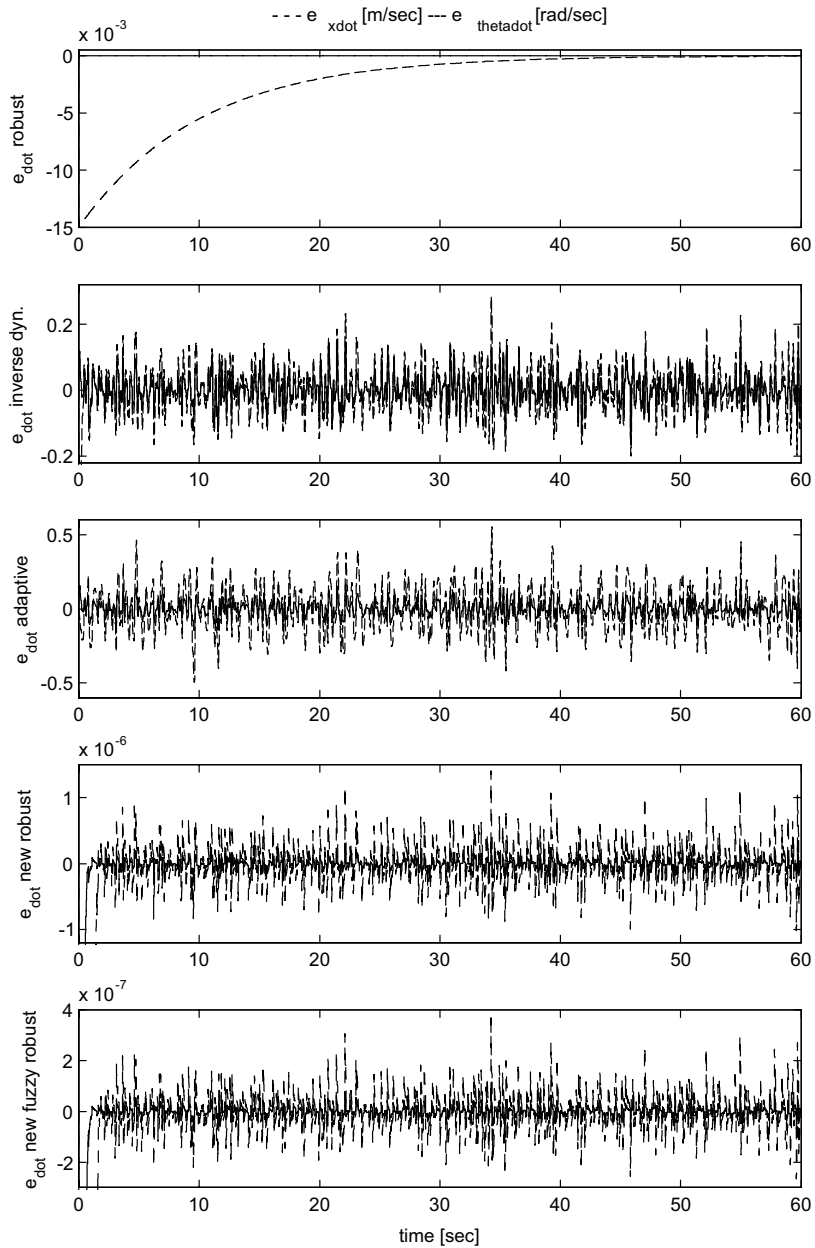


Fig. 12. Comparison of tracking error rates, Case 2 (constant parameters, added disturbance noise).

6.2.3. Case 3: varying parameters, no disturbance noise

In this case, no noise was added to the generalized forces. The mass of the prismatic link of the manipulator changed as shown in Fig. 13.

The comparison between the controllers proposed here and other designs is shown in Figs. 14 and 15. It was obtained that all controllers yielded similar control efforts as shown for Case 3 in

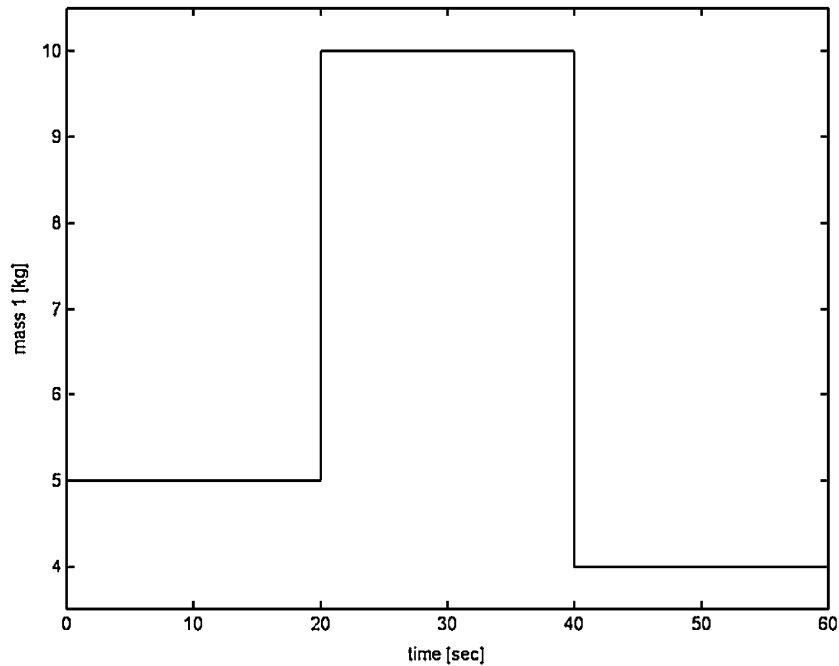


Fig. 13. Time history of the prismatic link mass.

Fig. 5. The largest tracking error changes corresponded to the displacement of the prismatic link due to its variable weight (see Fig. 14). The errors for the new robust controllers were of order 10^{-7} m, compared to displacement errors of up to 0.03 m for the inverse dynamics controller and up to 0.14 m for the adaptive controller. For the new non-fuzzy robust controller, the tracking rate errors for the prismatic link spike in the order of 10^{-6} m/s. The fuzzy version of the controller shows a jump in tracking rate error 50% smaller than the non-fuzzy version.

6.3. Fuzzy manifold robust controller

The fuzzy manifold robust controller (see Section 5.3) was implemented using Eqs. (71)–(74) and $Q_r(\|B\| \leq \varepsilon) \equiv s_\varepsilon$ as implemented for the fuzzy manifold-boundary robust controller (Shape III). No noise was added to the generalized forces, and the masses of the plant links remained constant throughout the simulations. The centers of the membership functions used for e_i were computed by setting the minimum and maximum limits of the error domains, and the number of membership functions (including the extremes):

$$\underline{e}_i^l = \underline{e}_i^1 + (i - 1) * \frac{e_i^{N_i} - e_i^1}{N_i}, \quad (98)$$

where $\underline{e}_1^1 = -0.25$ m, $\underline{e}_1^{21} = 0.25$ m, $\underline{e}_2^1 = -0.5$ rad, $\underline{e}_2^{21} = 0.5$ rad.

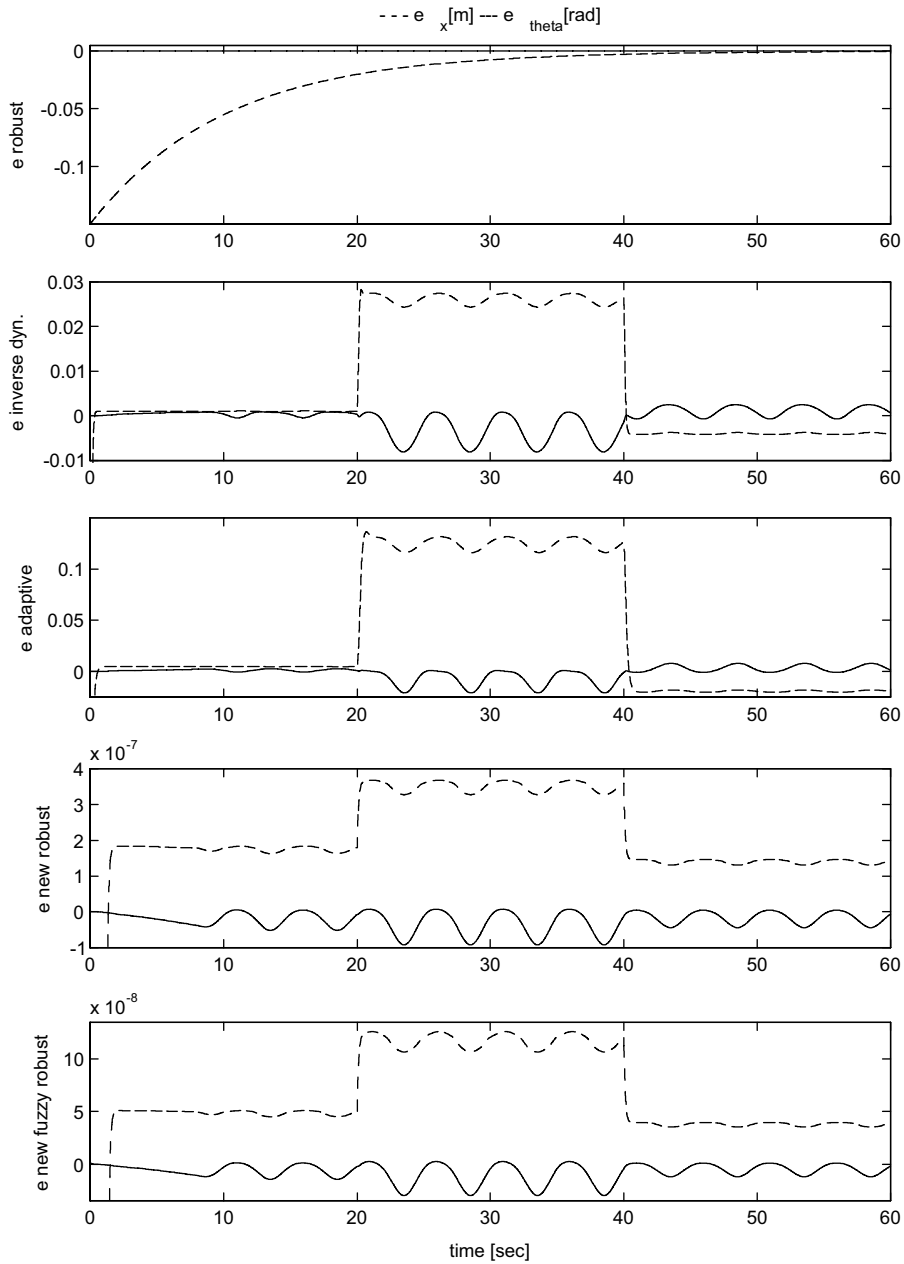


Fig. 14. Comparison of tracking errors, Case 3 (varying parameters, no disturbance noise).

Fig. 16 shows the results of the simulation. We see that the control effort was similar to that obtained for the new (non-fuzzy) robust controller but the tracking errors and tracking rate errors were *one order of magnitude smaller* (see Figs. 8 and 9).

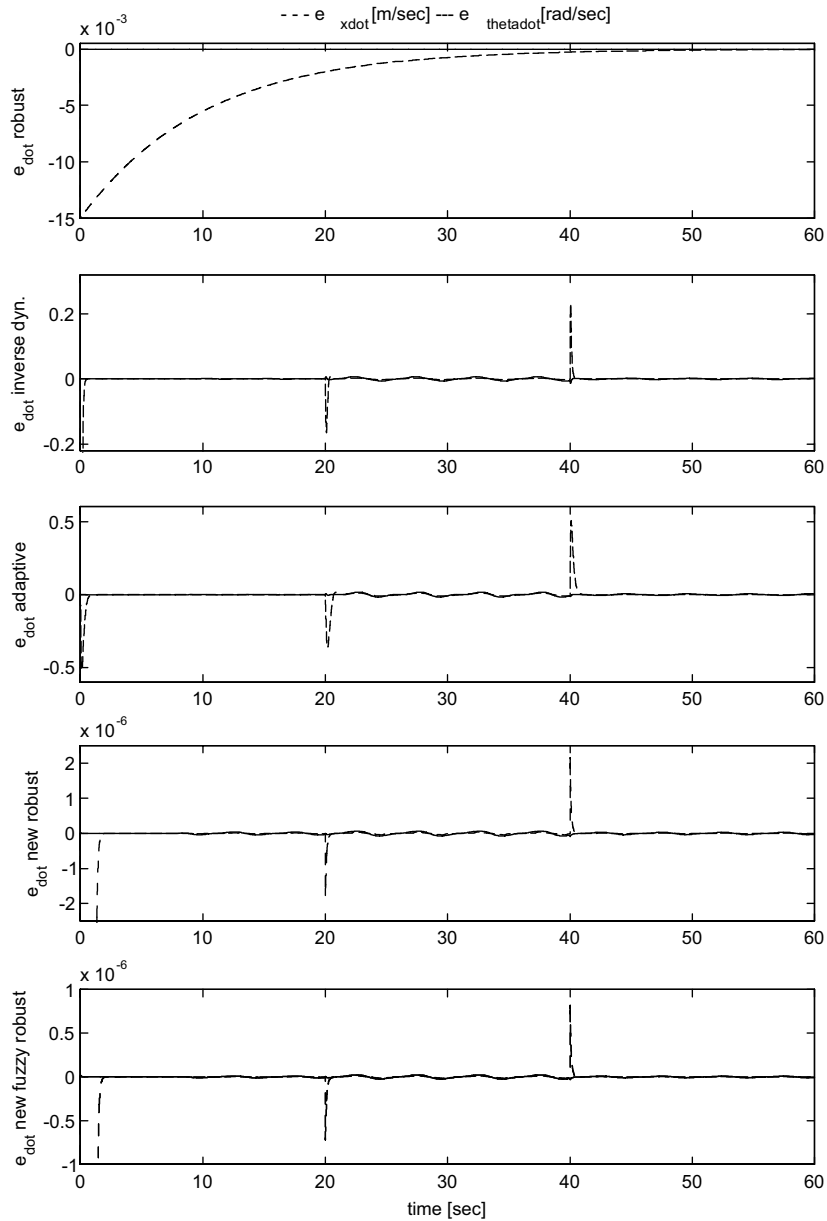


Fig. 15. Comparison of tracking error rates, Case 3 (varying parameters, no disturbance noise).

7. Conclusions

In this paper, an approach for the design of robust fuzzy control laws for a large class of mechanical systems was developed. Lyapunov's Stability Theory was used to ensure closed loop stability in the presence of plant perturbations, bounded disturbances and saturation. A methodology for the

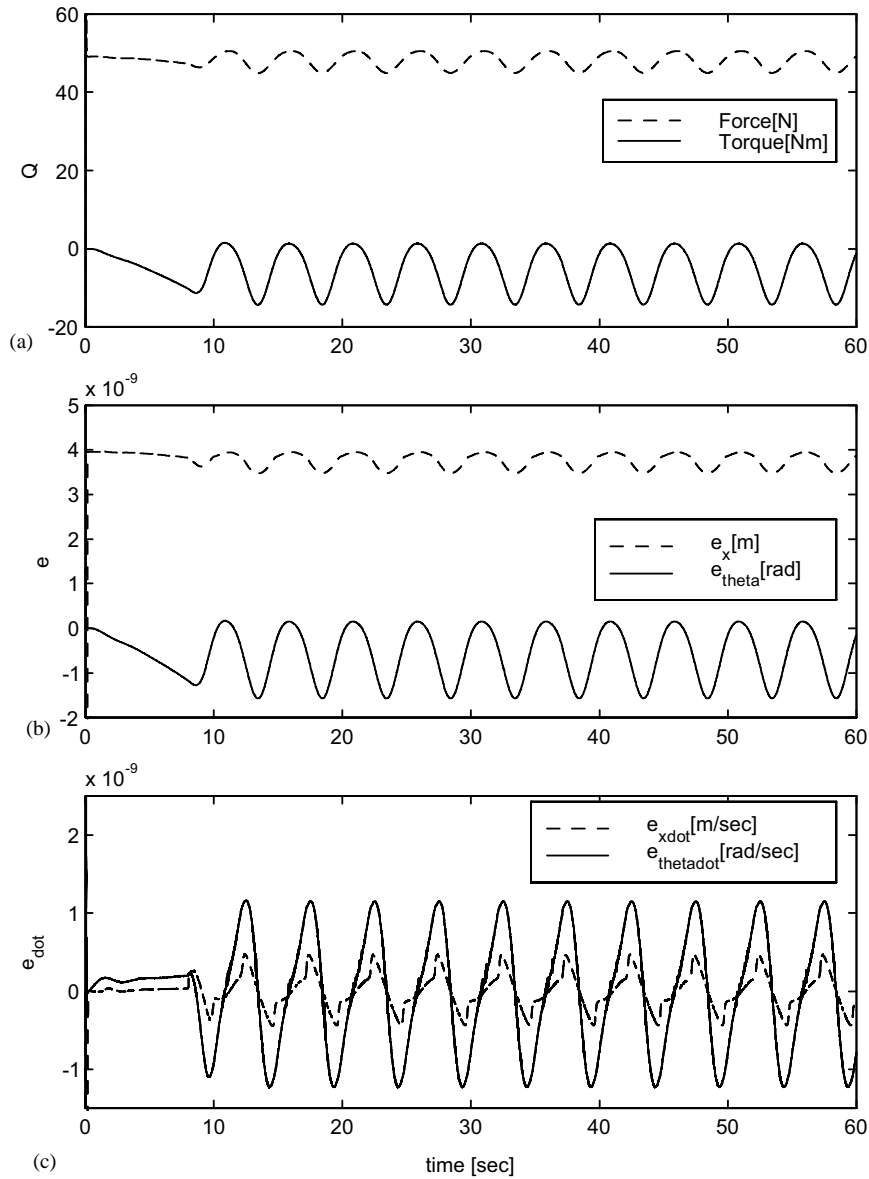


Fig. 16. Fuzzy manifold robust controller; (a) control effort, (b) tracking errors, (c) tracking error rates.

design of robust fuzzy control systems with guaranteed closed-loop stability was developed by applying the inherent properties of an important class of mechanical and aerospace systems such as robotic manipulators and large spacecraft.

The tracking control methodology is based on Lyapunov’s Stability Theory and its extensions due to Leitmann and Corless. Conditions were determined for robust stability and performance in the presence of plant uncertainties, bounded disturbances and saturation. These conditions involve a large

number of parameters and functional dependencies that can be chosen by the designer and, therefore, are well suited for Fuzzy Logic Control implementations. Two different fuzzy implementation methods of the proposed control system were analyzed and their relative advantages were discussed.

An extensive simulation study of the proposed approach was conducted which showed the superior performance of the proposed control methodology, compared to other robust control design techniques. We investigated the performance of the tracking control design methodology proposed in the paper. The implementations of the method, namely the fuzzy and non-fuzzy implementations, were simulated using MATLAB/Simulink. The performance of the new algorithms was compared to other robust control design methods, namely robust saturation-type, inverse dynamics and adaptive inertia-related.

The results demonstrate that the new robust controller design has no chattering, significantly smaller tracking errors and exhibits faster convergence compared to the other control laws evaluated in this paper. The fuzzy logic implementations of the new robust controller proved to be superior in tracking accuracy compared to non-fuzzy implementations.

Acknowledgements

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Appendix

Input-to-state stability [13]

Definition A.1. Consider the system

$$\dot{x} = f(t, x, u), \quad (\text{A.1})$$

where $f: [0, \infty) \times D \times D_u \rightarrow R^n$ is piecewise continuous in t and locally Lipschitz in x and u , $D \subset R^n$ is a domain that contains $x=0$, and $D_u \subset R^m$ is a domain that contains $u=0$. The input $u(t)$ is a piecewise continuous, bounded function of t for all $t \geq 0$. The system is said to be locally input-to-state stable if there exist a class \mathcal{KL} function β , a class \mathcal{K} function γ , and positive constants k_1 and k_2 such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any input $u(t)$ with $\sup_{t \geq t_0} \|u(t)\| < k_2$, the solution $x(t)$ exists and satisfies:

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma \left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\| \right) \quad (\text{A.2})$$

for all $t \geq t_0 \geq 0$. It is said to be input-to-state stable if $D = R^n$, $D_u = R^m$, and inequality (A.2) is satisfied for any initial state $x(t_0)$ and any bounded input $u(t)$.

Theorem A.2. Let $D = \{x \in R^n / \|x\| < r\}$, $D_u = \{u \in R^m / \|u\| < r_u\}$, and $f: [0, \infty) \times D \times D_u \rightarrow R^n$ be piecewise continuous in t and locally Lipschitz in x and u . Let $V: [0, \infty) \times D \rightarrow R$ be a continuously

differentiable function such that:

$$\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|), \quad (\text{A.3})$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq -\alpha_3(\|x\|), \quad \forall \|x\| \geq \rho(\|u\|) > 0 \quad (\text{A.4})$$

$\forall (t, x, u) \in [0, \infty) \times D \times D_u$ where $\alpha_1, \alpha_2, \alpha_3$ and ρ are class \mathcal{K} functions. Then, the system $\dot{x} = f(t, x, u)$ is locally input-to-state stable with $\gamma = \alpha_1^{-1} \cdot \alpha_2 \cdot \rho$, $k_1 = \alpha_2^{-1}(\alpha_1(r))$ and $k_2 = \rho^{-1}(\min\{k_1, \rho(r_u)\})$. Moreover, if $D = \mathbb{R}^n$, $D_u = \mathbb{R}^m$, and α_1 is a class \mathcal{K}_∞ function, then the system $\dot{x} = f(t, x, u)$ is input-to-state stable with $\gamma = \alpha_1^{-1} \cdot \alpha_2 \cdot \rho$.

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