Towards a Meaningful Fuzzy Analysis of Urbanistics Data

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ABSTRACT — The author presents a method of analyzing and interpreting data from an Urbanistics project. A fuzzy hassefication technique is used in the preliminary analysis. Then an algorithm that compares Hasse diagrams is also used. The resulting mappings are evaluated for their "goodness of fit."

I. Introduction

Society, just as humankind, has been evolving into a more complex, dynamic system. The urban environment, in particular, is always plagued with problems and dissatisfied inhabitants. In this paper, the author develops a method for analyzing and interpreting interview data from inhabitants of a city. The data is in the form of mathematical relations. The purpose of the investigation was to eventually design a consultative system for urban problems.

The standard way of processing relations is by extracting deep structure from them. This may be achieved by obtaining certain *closures* and *interiors* depending on the relational properties under investigation. Perhaps one of the most interesting group of properties are those exhibited by *orders* and *preorders*. Normally, Hasse diagrams are used to graphically illustrate the different categories hidden by such relations.

In [4], the author studied the applicability of various relational strength measures when comparing and analyzing Hasse diagrams. The Hasse diagrams used were derived from data collected in an Urbanistics project [3] that investigates human perception of an urban environment. These diagrams represent certain knowledge structures that the interviewee possesses. In this paper, the author elaborates on and continues with the results presented in [4].

Perhaps one of the most interesting aspects that have to be considered when comparing Hasse diagrams is the fact that the reference sets are actually (disjoint) partitions of the original reference sets. Hence, each node on a Hasse diagram may constitute, for example, an equivalence class. In [4], it was proposed that a homomorphism between the nodes in the diagram be established prior to comparing such structures. This was then used as a "guide" in the actual comparison. Nodes were mapped based on (the degree of) containment or on (the degree of) overlap. Structure mappings may then be established based on these node-mappings.

II. Processing Methodology Used

The data used in [4] was processed using a conceptual procedure developed by Bandler and Kohout [1]. Given some fuzzy relation $T \subseteq X \times Y$, fuzzy local preorders (say, $R \subseteq X \times X$) were derived by taking the local preorder closure of the triangle product, $T \triangleleft T^{-1}$. Only the Kleene-Dienes fuzzy implication operator, defined as:

$$a \rightarrow b \quad = \quad \bar{a} \vee b \qquad (1)$$

was used in [4]. The following *Hassefication procedure*, from [1], was then used

1. Take an α-cut R_α.
2. Form $S = \text{sym Int } R_\alpha$. This relation is a *local equivalence*.
3. Remove the zero-class C_0, consisting of all x's unrelated by S to any elements.
4. Let E be the *factor set* of $X \setminus C_0$ according to the relation S.
5. Denote by $\preceq$ the *factor relation* R_α/S, which is an *order*.
6. Let $P = (E, \preceq)$. Draw the corresponding Hasse diagram $H(P)$.

III. Sample of Data Used

Relations in [4] were obtained from three respondents chosen at random (referred to in the study as gr17, gr18, and gr21). Various alpha-cuts were used on these relations to generate 14 different Hasse diagrams. Of these, six were chosen for the actual investigation, two from each respondent (the reasoning behind the choices are discussed in [4]). Three of those are presented in this section. In what follows, "H_{ij}" denotes the Hasse diagram for gri at an alpha-cut of α=j. So, $H_{18,0.60}$ denotes a Hasse diagram for *gr18* at α=0.60.

1. Hasse diagram, $H_{18,0.60}$

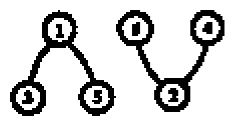

Figure 1 $H_{18,0.60}$. Hasse diagram for *gr18* at α=0.60.

One of the six Hasse diagrams considered is depicted in Figure 1. The node labels for this graph are given in Table 1 below.

Table 1 Interpretation of node labels for $H_{18,0.60}$

Label Used	*URBS Label*	*Conceptual Interpretation*
1	{3, 4, 12, 14, 15}	{well-maintained, friendly, racially relaxed, desirable, safe}
2	{3n, 4n, 12n, 14n, 15n}	{run-down, unfriendly, racially tense, undesirable, dangerous}
3	{5}	{beautiful}
4	{5n}	{ugly}
5	{10}	{feel you can influence}
6	{10n}	{feel you cannot influence}

In the column labeled *URBS Label*, the numbers indicate construct numbers and the *n* represents the construct's negative pole. This comes from the knowledge elicitation technique used in the Urbanistics project [3]. The interviewees were allowed to relate bipolar constructs, containing both positive and negative poles, with tangible landmarks in the vicinity of their neighborhood.

2. Hasse diagram, $H_{18,0.80}$

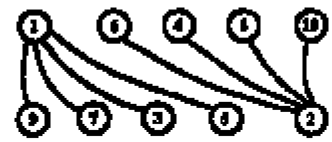

Figure 2 $H_{18,0.80}$. Hasse diagram for *gr18* at α=0.80.

The next Hasse diagram considered is depicted in Figure 2. The node labels for this graph are given in Table 2 below.

Table 2 Interpretation of node labels for $H_{18,0.80}$

Label Used	*URBS Label*	*Conceptual Interpretation*
1	{3, 12, 15}	{well-maintained, racially relaxed, safe}
2	{3n, 12n, 15n}	{run-down, racially tense, dangerous}
3	{4}	{friendly}
4	{4n}	{unfriendly}
5	{5}	{beautiful}
6	{5n}	{ugly}
7	{10}	{feel you can influence
8	{10n}	{feel you cannot influence}
9	{14}	{desirable}
10	{14n}	{undesirable}

3. Hasse diagram, $H_{21,0.50}$

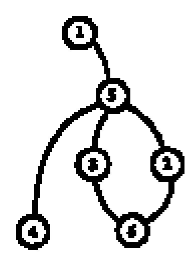

Figure 3 $H_{21,0.50}$. Hasse diagram for *gr21* at α=0.50.

Another one of the diagrams considered is depicted in Figure 3. The node labels for this graph are given in Table 3 below.

Table 3 Interpretation of node labels for $H_{18,0.80}$

Label Used	*URBS Label*	*Conceptual Interpretation*
1	{3}	{well-maintained}
2	{4, 10}	{friendly, feel you can influence}
3	{5}	{beautiful}
4	{12}	{racially relaxed}
5	{14}	{desirable}
6	{15}	{safe}

IV. Sample Mappings

A methodology for developing homomorphisms between Hasse diagrams is outlined in [4]. This technique defines the mappings as a pair $\mathscr{F} = \langle f, g \rangle$ consisting of a node and a path mapping, respectively. Mapping details are outlined in [4].

The mappings in this paper were analyzed in two ways. First, mappings between Hasse diagrams from the same respondent were derived and analyzed. The only difference between these diagrams is that they were obtained from distinct alpha-cuts. Next, mappings between Hasse diagrams from different respondents were derived and analyzed. The scalar cardinality (or $\sum Count$)

$$\Sigma Count\,(R) \quad = \quad \sum_{i,j} \rho_R\,(x_i\,,\,x_j) \qquad (2)$$

and "mean" cardinality

$$\bar{R} \quad = \quad \frac{\Sigma Count\,(R)}{|X|^2} \qquad (3)$$

of the mappings f and g were computed for comparison. These are presented in tabular form.

The choice of the alpha values and the methodology for defining the mappings f and g are beyond the scope of the current paper. Two of the mappings considered are presented next.

1. Mapping between $H_{18,0.85}$ and $H_{18,0.80}$

The Hasse diagram mapping depicted in Figure 4 is from *gz18*. The nodes map f is a *one-to-many* mapping because the node set of $H_{18,0.80}$ is a subset of the node set of $H_{18,0.85}$. This, plus the fact that the two diagrams have the same number of strata, results in a good path map g.

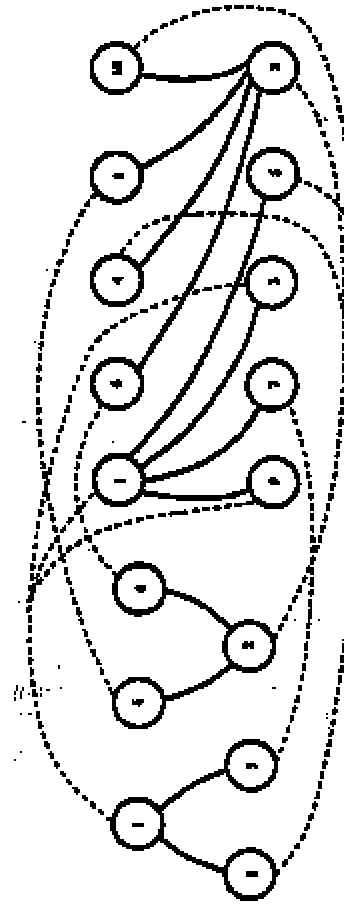

Figure 4 Mapping $H_{18,0.80}$ and $H_{18,0.85}$.

The scalar and "mean" measures for this mapping are shown in Table 4. These values correspond to the cardinalities of the factor set and factor relation of $H_{18,0.80}$.

Table 4 Cardinalities of mappings between $H_{18,0.85}$ and $H_{18,0.80}$

$\mathscr{F}: H_{18,0.85} \rightarrow H_{18,0.80}$ where $\mathscr{F} = \langle f, g \rangle$	*Scalar Cardinality*	*Mean Cardinality*
Nodes Map, f	10	0.166
Paths Map, g	18	0.100

2. Mapping between $H_{18,0.80}$ and $H_{21,0.50}$

The mapping derived between $H_{18,0.80}$ and $H_{21,0.50}$ is depicted in Figure 5. $H_{18,0.80}$ is a disconnected graph. Nodes in one of its connected components were not matched with $H_{21,0.50}$ because this component involved negative constructs only. All nodes in $H_{21,0.50}$ involve positive constructs.

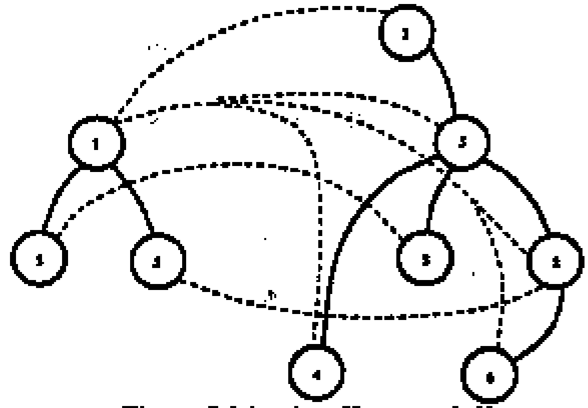

Figure 5 Mapping $H_{18,0.80}$ and $H_{21,0.50}$.

The scalar and "mean" measures for this mapping are shown in Table 5.

Table 5 Cardinalities of mappings between $H_{18,0.80}$ and $H_{21,0.50}$

$\mathcal{F}: H_{18,0.80} \to H_{21,0.50}$ where $\mathcal{F} = (f, g)$	*Scalar Cardinality*	*Mean Cardinality*
Nodes Map, f	7	0.194
Paths Map, g	19	0.139

The larger scalar cardinality for f, considering the fact that only one of the two components of $H_{18,0.80}$ are mapped, is attributed to the larger class size of the nodes in $H_{18,0.80}$ (in particular, refer to the node labelled "1" in Figure 5. The computed scalar cardinality for g is interesting because it is greater than the scalar cardinalities of the factor relations of $H_{18,0.80}$ and $H_{21,0.50}$. This is due to the fact that f is a *one-to-many* mapping: some nodes in $H_{18,0.80}$ are mapped to more than one node in $H_{21,0.50}$; unlike the previous mapping where the scalar cardinality of g was between the scalar cardinalities of the factor relations of the Hasse diagrams involved.

V. Summary and Conclusions

Between the six Hasse diagrams considered in this investigation, nine mappings were carefully analyzed. This involved computing and relating the pairs of (scalar) cardinalities: that of the *original* factor sets and factor relations, and that from the *induced* directed subgraphs. An estimate of the "goodness of fit" of each mapping, based on [4], was computed. This uses the proportion of nodes and links actually mapped by the proposed algorithm. Hence, node and path mappings were evaluated based on the ratio between the cardinality of the induced factor set (or factor relation) and the original factor set (or factor relation).

The significance of the *congruence* between the aspatial and spatial structures that constitute knowledge representation of the urban environment is discussed in [5]. Consequently, it is important to develop a method that generates a meaningful interpretation of some of our higher mental processes. These formulations are a step towards that direction.

References

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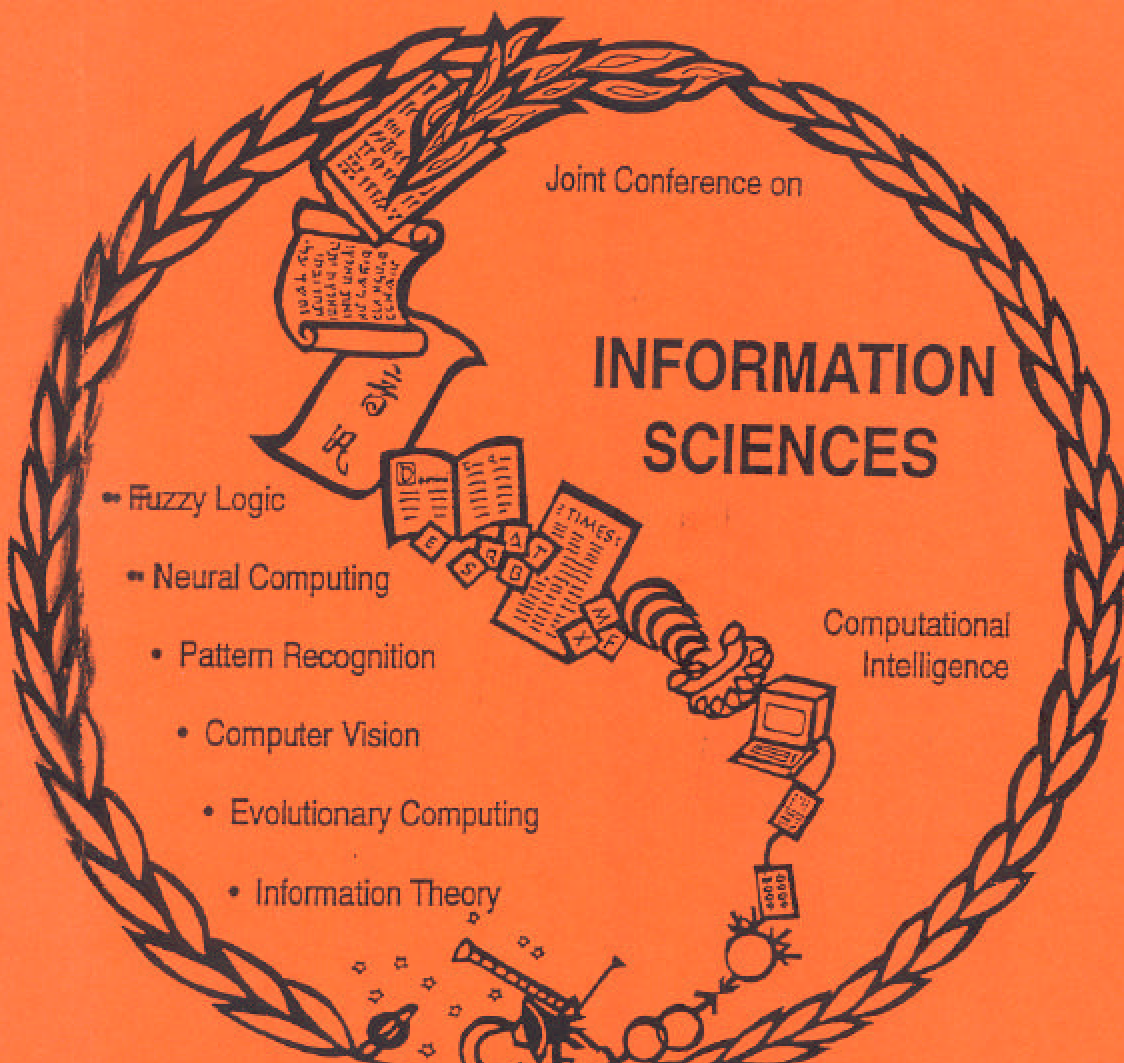

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