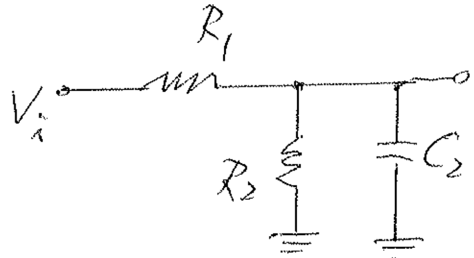
# EECE-315 Exercise # 6

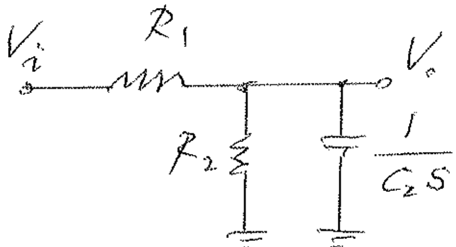
7.3




$R_1 = 10\ k\Omega$, $R_2 = 20\ k\Omega$

$C_2 = 10\ \mu F$

(a)




$$R_2 \parallel \frac{1}{C_2 s} = \frac{R_2 \times \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

$$V_o = V_i \times \frac{\frac{R_2}{R_2 C_2 s + 1}}{R_1 + \frac{R_2}{R_2 C_2 s + 1}} = V_i \times \frac{R_2}{R_1 R_2 C_2 s + R_1 + R_2}$$

$$\therefore H(s) = T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_1 R_2 C_2 s + R_1 + R_2} = \frac{20}{2s + 30} = \underline{\underline{\frac{10}{s+15}}}$$

(b) $\tau = (R_1 \parallel R_2) \times C_2 = \left(\frac{R_1 R_2}{R_1 + R_2}\right) \times C_2 = \frac{1}{15} = \underline{\underline{66.7\ msec}}$

(c) $$H(j\omega) = H(j2\pi f) = \frac{10}{j2\pi f + 15} = \frac{10}{15\left(1 + j\frac{2\pi f}{15}\right)}$$

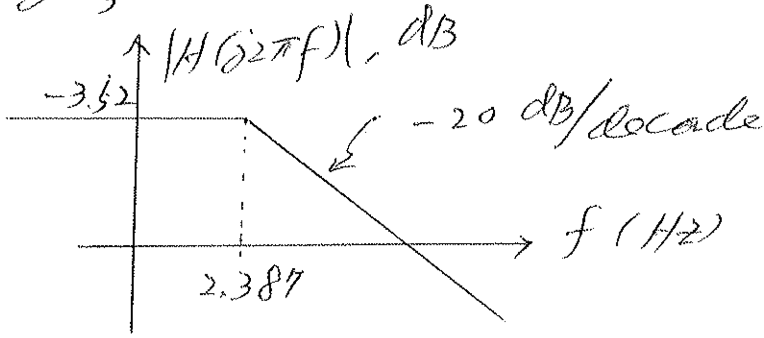
$$H(j2\pi f) = \frac{\frac{2}{3}}{1 + j\frac{f}{f_1}} \quad \text{where } f_1 = \frac{15}{2\pi} = 2.387\ Hz$$

Corner freq. $= \underline{\underline{2.387\ Hz}}$

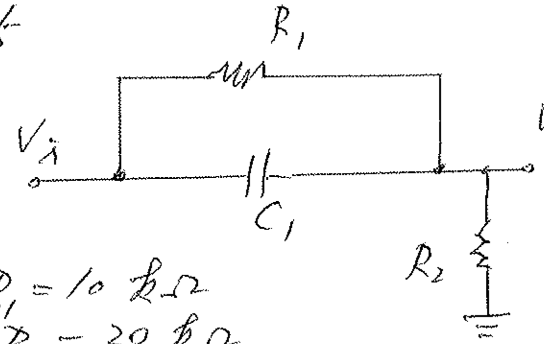
(d) $$|H(j2\pi f)| = \frac{\frac{2}{3}}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}} \quad ; \quad |H(j2\pi f)|_{f=0} = \frac{2}{3}$$

$$|H(j2\pi f)|_{f=\infty} = 0$$

$20 \log\left(\frac{2}{3}\right) = -3.52\ dB$



7.5




$R_1 = 10\ k\Omega$
$R_2 = 20\ k\Omega$
$C_1 = 10\ \mu F$

$$R_1 \parallel \frac{1}{C_1 S} = \frac{R_1 \times \frac{1}{C_1 S}}{R_1 + \frac{1}{C_1 S}} = \frac{R_1}{R_1 C_1 S + 1}$$

$$V_o = V_i \times \frac{R_2}{R_2 + \left(\frac{R_1}{R_1 C_1 S + 1}\right)}$$

$$H(s) = \frac{V_o}{V_i} = \frac{R_2 (R_1 C_1 S + 1)}{R_2 (R_1 C_1 S + 1) + R_1} = \underline{\underline{\frac{R_1 R_2 C_1 S + R_2}{R_1 R_2 C_1 S + (R_1 + R_2)}}}$$

$$H(j\omega) = \frac{j\omega R_1 R_2 C_1 + R_2}{j\omega R_1 R_2 C_1 + (R_1 + R_2)}$$

(a) $$H(j\omega)\Big|_{\omega=0} = \frac{R_2}{R_1 + R_2} = \underline{\underline{\frac{2}{3}}}$$

(b) $$H(j\omega)\Big|_{\omega=\infty} = \underline{\underline{1}}$$

(c) $$H(s) = T(s) = \frac{2S + 20}{2S + 30} = \underline{\underline{\frac{S + 10}{S + 15}}}$$

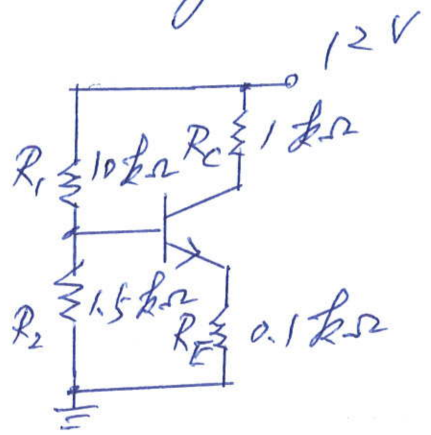
(d) $$T(s) = \frac{10\left(1 + \frac{S}{10}\right)}{15\left(1 + \frac{S}{15}\right)} = \underline{\underline{\frac{2}{3} \times \frac{1 + \frac{1}{10} \cdot S}{1 + \frac{1}{15} \cdot S}}}$$

$$K = \frac{2}{3} \quad \left( = H(j\omega)\Big|_{\omega=0} \right)$$

$$\tau_A = \frac{1}{10} = 0.1 \text{ sec}$$

$$\tau_B = \frac{1}{15} = 66.7 \text{ msec}$$

7.17 dc analysis to determine $I_{CQ}$.



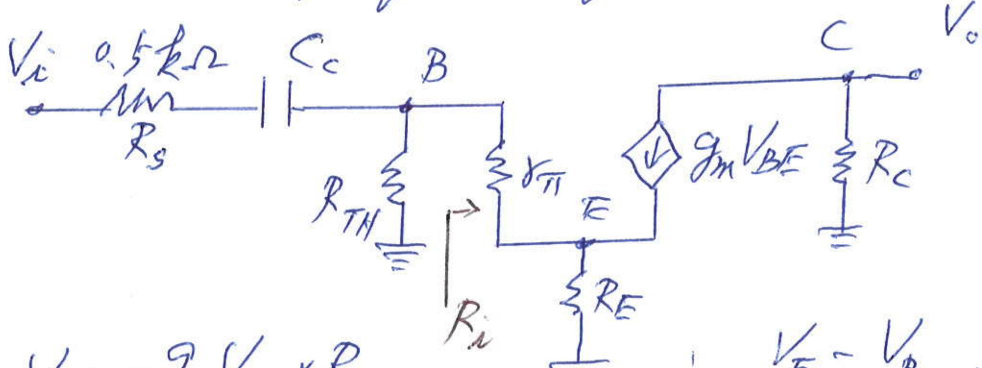

$$-12 \times \frac{1.5}{10+1.5} + \frac{I_{CQ}}{\beta} \times R_{TH} + 0.7 + \left(\frac{I_{CQ}}{\beta} + I_{CQ}\right) \times R_E = 0$$

$$R_{TH} = R_1 \parallel R_2 = 1.3\ k\Omega$$

$$\underline{I_{CQ} = 7.59\ mA}$$

$$g_m = \frac{I_{CQ}}{0.025} = \underline{303.6\ mA/V}, \quad r_\pi = \frac{\beta}{g_m} = \underline{329.4\ \Omega}$$

ac low-freq. analysis




$$V_o = -g_m V_{BE} \times R_C$$

$$\underline{V_o = -303.6\, V_{BE}} \quad \cdots ①$$

$$\frac{V_B}{R_{TH}} + \frac{V_{BE}}{r_\pi} + \frac{V_B - V_i}{R_S + \frac{1}{C_c S}} = 0$$

$$\left(\frac{1}{R_{TH}} + \frac{1}{R_S + \frac{1}{C_c S}}\right) V_B + \frac{V_{BE}}{r_\pi} = \frac{V_i}{R_S + \frac{1}{C_c S}} \quad \cdots ②$$

$$\frac{V_E - V_B}{r_\pi} + \frac{V_E}{R_E} - g_m V_{BE} = 0$$

$$-\frac{V_{BE}}{r_\pi} - g_m V_{BE} + \frac{V_E}{R_E} = 0$$

$$\therefore \underline{V_E = \left(\frac{1}{r_\pi} + g_m\right) R_E V_{BE}}$$

$$V_B = V_{BE} + V_E$$

$$\underline{V_B = \left(1 + \frac{R_E}{r_\pi} + g_m R_E\right) V_{BE}} \quad \cdots ③$$

$$V_B = 31.66\, V_{BE}$$

7.7 Combining eqs ①, ② and ③, we obtain

$$A_v = H(s) = \frac{V_o}{V_i} = -\frac{11080 C_c S}{1 + 1655.5 C_c S}, \quad A_{V(MB)} = -\frac{11080}{1655.5} = -6.69$$

$$f_L = \frac{1}{2\pi \times (1655.5 \times C_c)} = \underline{\underline{961.4 \text{ Hz}}},$$

<u>Another method</u>

Since $C_c$ is the only capacitor, we can determine the $f_L$ by the time constant due to $C_c$.

$$R_i = r_\pi + (1+\beta) R_E = 10.43 \text{ k}\Omega$$

$$R_S + (R_{TH} \parallel R_i) = 1.66 \text{ k}\Omega$$

$$\therefore \quad f_L = \frac{1}{2\pi \times 1.66 \times 10^3 \times 0.1 \times 10^{-6}} = \underline{\underline{958.8 \text{ Hz}}}$$

$$A_{V,MB} = -\frac{g_m R_C}{1 + g_m R_E} \times \frac{R_i \parallel R_{TH}}{R_S + (R_{TH} \parallel R_i)}$$

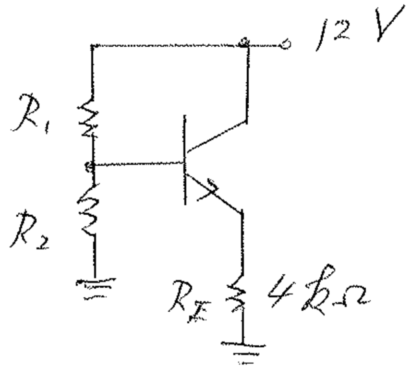
$$A_{V,MB} = \underline{\underline{-6.77}}$$

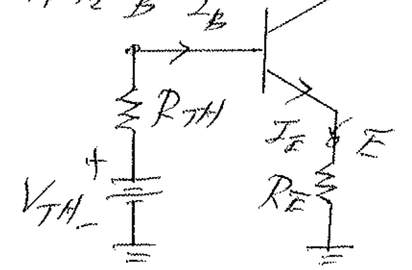
7.21

(a) dc analysis

$R_{TH} = R_1 \parallel R_2$

$V_{TH} = 12 \times \frac{R_2}{R_1 + R_2}$





$\beta = 120$

$I_{EQ} = 1.5$ mA

$V_{BE} = 0.7$ V

Let $(\beta+1)R_E = 10 R_{TH}$ , $\therefore R_{TH} = \frac{121}{10} \times 4 = 48.4$ kΩ

$$-V_{TH} + I_B \times R_{TH} + V_{BE} + I_E \times R_E = 0$$

$$-12 \times \frac{R_1 R_2}{R_1 + R_2} \times \frac{1}{R_1} + \frac{I_E}{\beta+1} \times R_{TH} + 0.7 + R_E I_E = 0$$

$$-12 \times \frac{R_{TH}}{R_1} + \left(\frac{R_{TH}}{\beta+1} + R_E\right) \times I_E + 0.7 = 0$$

$$\left(\frac{48.4}{121} + 4\right) \times 1.5 + 0.7 = \frac{12 \times 48.4}{R_1}$$

$\therefore$

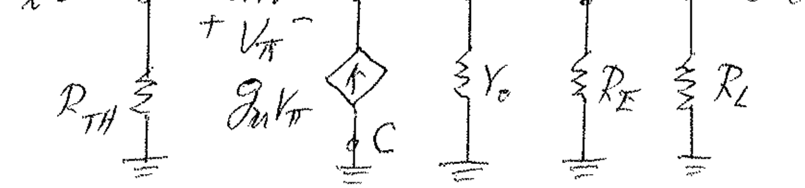
$$R_1 = \frac{580.8}{7.3} = 79.56 \text{ k}\Omega \, . \quad \frac{R_1 R_2}{R_1 + R_2} = R_{TH}$$

$$R_2 = \frac{R_1 \times R_{TH}}{R_1 - R_{TH}} = 123.6 \text{ k}\Omega$$

(b) $g_m = \frac{I_{CQ}}{0.025} = \frac{I_{EQ} \times \frac{\beta}{\beta+1}}{0.025} = 59.5$ mA/V

$r_\pi = \frac{\beta}{g_m} = 2.017$ kΩ , $r_o = \frac{V_A}{I_{CQ}} = \frac{50}{I_{EQ} \times \frac{\beta}{\beta+1}} = 33.61$ kΩ

ac equivalent circuit (midband)



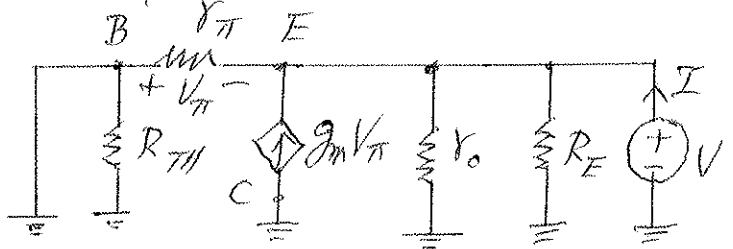
$r_o \parallel R_E \parallel R_L = 1.888$ kΩ

7.21 (b) $\dfrac{V_o}{1.888} - g_m V_\pi - \dfrac{V_\pi}{r_\pi} = 0$ ; $V_\pi = V_i - V_o$

$\therefore \dfrac{V_o}{1.888} - \left(g_m + \dfrac{1}{r_\pi}\right) \times (V_i - V_o) = 0$

$$A_v = \frac{V_o}{V_i} = \frac{g_m + \frac{1}{r_\pi}}{\frac{1}{1.888} + \left(g_m + \frac{1}{r_\pi}\right)} = \underline{\underline{0.991}}$$

(c) equivalent circuit to determine $R_O$



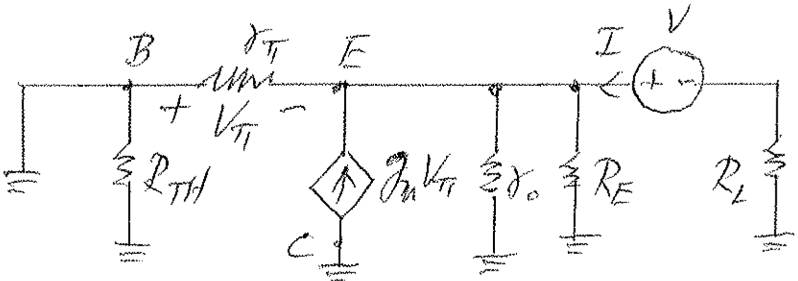

$R_o \equiv \dfrac{V}{I}$

$I = \dfrac{V}{R_E} + \dfrac{V}{r_o} - g_m V_\pi - \dfrac{V_\pi}{r_\pi}$ ; $V_\pi = 0 - V = -V$

$\therefore I = \left(\dfrac{1}{R_E} + \dfrac{1}{r_o} + g_m + \dfrac{1}{r_\pi}\right) V = 60.28\,V$

$R_o = \dfrac{V}{I} = 0.01659\ k\Omega = \underline{\underline{16.59\ \Omega}}$

(d) Short-circuit time constant for $C_{C_2}$




$R_{EQ} \equiv \dfrac{V}{I}$

$-I + \dfrac{V - IR_L}{R_E} + \dfrac{V - IR_L}{r_o} - g_m V_\pi - \dfrac{V_\pi}{r_\pi} = 0$ ; $V_\pi = 0 - (V - IR_L) = IR_L - V$

$-\left(1 + \dfrac{R_L}{R_E} + \dfrac{R_L}{r_o}\right) I + \left(\dfrac{1}{R_E} + \dfrac{1}{r_o}\right) V - \left(g_m + \dfrac{1}{r_\pi}\right)(IR_L - V) = 0$

$$\therefore R_{EQ} = \frac{V}{I} = \frac{1 + \frac{R_L}{R_E} + \frac{R_L}{r_o} + \left(g_m + \frac{1}{r_\pi}\right) R_L}{\frac{1}{R_E} + \frac{1}{r_o} + g_m + \frac{1}{r_\pi}} = \underline{\underline{4.017\ k\Omega}}$$

$\tau = R_{EQ} \times C_{C_2} = \underline{\underline{8.034\ msec}}$  $f_L = \dfrac{1}{2\pi\tau} = \underline{\underline{19.8\ Hz}}$

D7.30 <u>dc analysis</u>

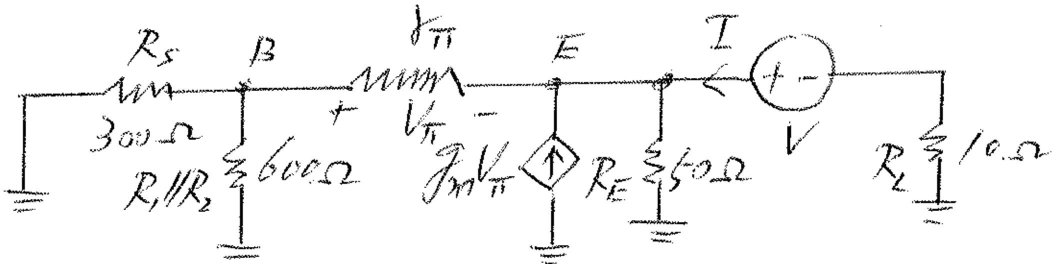
$$-5 \times \frac{R_2}{R_1+R_2} + 0.7 + \frac{R_1 R_2}{R_1+R_2} \times I_B + R_E I_E = 0$$

$$-2.5 + 0.7 + 0.6 \times \frac{I_C}{100} + \frac{101}{100} I_C \times 0.050 = 0$$

$\therefore$ $\underline{I_C = 31.9 \text{ mA}}$.

$$g_m = \frac{I_C}{0.025} = 1.276 \text{ A/V}, \quad r_\pi = \frac{\beta}{g_m} = 78.4\ \Omega$$

<u>Short-circuit Time Constant for $C_{C2}$</u>




$600 \| 300 = 200\ \Omega$, $\quad V_E = V - I R_L = V - 10 I \quad \cdots (1)$

$$\frac{V}{50} - I - g_m V_\pi + \frac{V_E}{78.4 + 200} = 0 \quad \cdots (2)$$

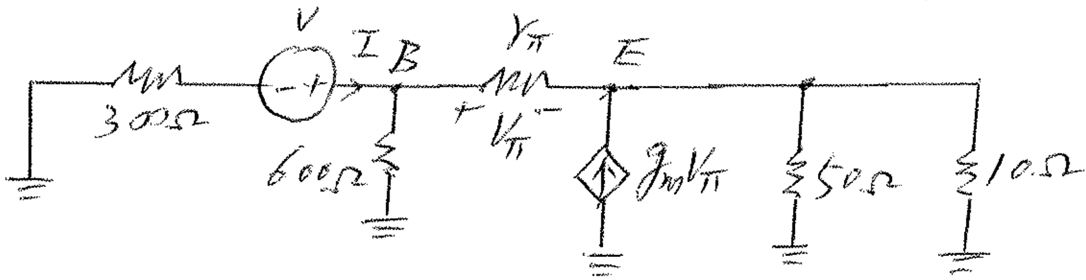
$$V_\pi = -V_E \times \frac{78.4}{78.4+200} = -0.282(V - 10I) \quad \cdots (3)$$

Combining eqs. (1), (2) and (3), we obtain

$$\frac{V}{I} = 11.31\ \Omega, \quad \frac{1}{11.31 \times C_{C2}} = 2\pi \times 25$$

$\therefore$ $\underline{C_{C2} = 562\ \mu F}$

D7.30 <u>Short-circuit Time constant for $C_{c1}$</u>




$50 \parallel 10 = 8.33\ \Omega$

$V_B = V - 300I \cdots ①, \quad V_\pi = V_B - V_E \cdots ②$

$$\frac{V_B}{600} - I + \frac{V_\pi}{r_\pi} = 0 \quad \cdots ③$$

$$\frac{V_E}{50 \parallel 10} - g_m V_\pi - \frac{V_\pi}{r_\pi} = 0 \quad \cdots ④$$

Combining eqs. ①, ②, ③ and ④, we obtain

$$\frac{V}{I} = 663\ \Omega$$

$663 \times C_{c1} = 100 \times 11.31 \times C_{c2}$

$$\therefore\ C_{c1} = \frac{100 \times 11.31 \times 562}{663} = 959\ \mu F$$

6.76 dc analysis

(a)

$$-12 \times \frac{12.7}{12.7+67.3} + \frac{12.7\times 67.3}{12.7+67.3} \times \frac{I_{CQ1}}{120} + 0.7 + \frac{121}{120} I_{CQ_1} \times 2 = 0$$

$\therefore\ I_{CQ_1} = 0.572\ mA$

$$-12 \times \frac{45}{45+15} + \frac{45\times 15}{45+15} \times \frac{I_{CQ2}}{120} + 0.7 + \frac{121}{120} I_{CQ_2} \times 1.6 = 0$$

$I_{CQ_2} = 4.86\ mA$

$g_{m_1} = \frac{I_{CQ1}}{0.025} = 22.88\ mA/V, \quad r_{\pi 1} = \frac{120}{g_{m_1}} = 5.245\ k\Omega$

$g_{m_2} = \frac{I_{CQ2}}{0.025} = 194.4\ mA/V, \quad r_{\pi 2} = \frac{120}{g_{m_2}} = 0.6173\ k\Omega$

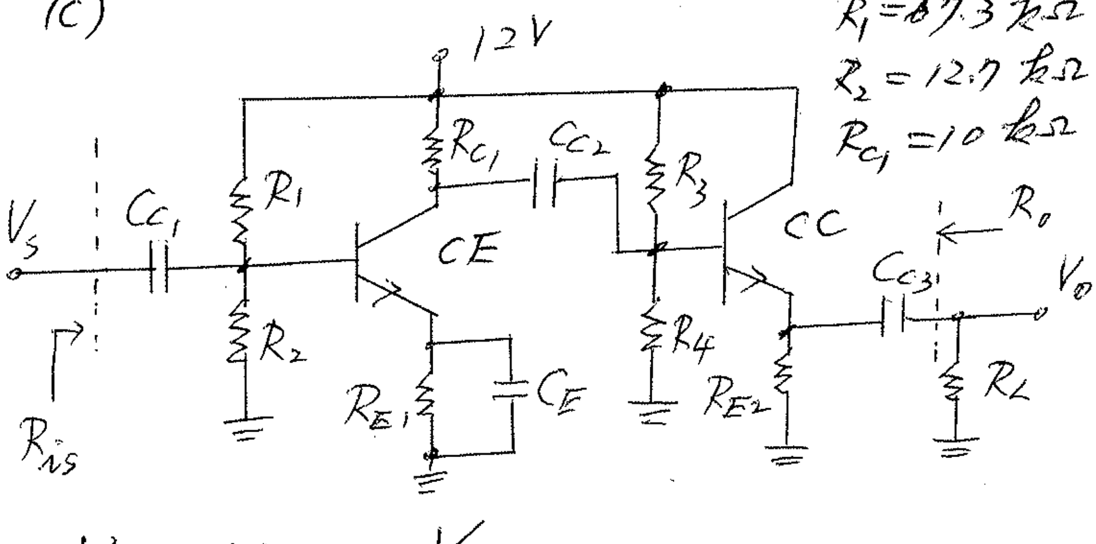
$r_{o1} = r_{o2} = \infty \quad (\because V_A = \infty)$

(c)

$R_1 = 67.3\ k\Omega$

$R_2 = 12.7\ k\Omega$

$R_{C_1} = 10\ k\Omega, R_{E1} = 2\ k\Omega$




$R_3 = 15\ k\Omega$

$R_4 = 45\ k\Omega$

$R_{E2} = 1.6\ k\Omega$

$R_L = 250\ \Omega$

$$\frac{V_o}{V_s} = \frac{V_o}{V_{b2}} \times \frac{V_{b2}}{V_s}$$

6.76

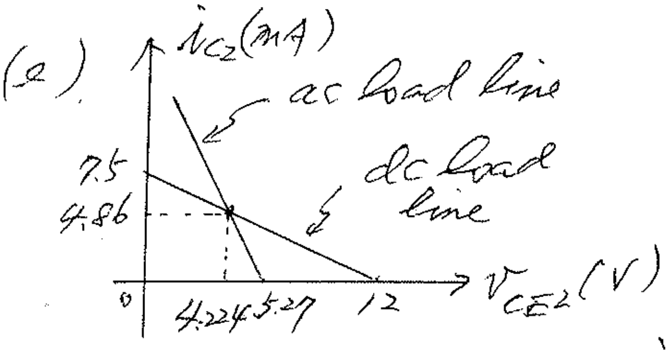
$$\frac{V_o}{V_{b2}} = \frac{g_{m2} + \frac{1}{r_{\pi 2}}}{g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{R_{E2} \parallel R_L}} = \underline{\underline{0.999}}$$

$$\frac{V_{b2}}{V_s} = -g_{m1} \times \left\{ R_{C1} \parallel R_3 \parallel R_4 \parallel \left[ r_{\pi 2} + (1+\beta_2)(R_{E2} \parallel R_L) \right] \right\} = -101$$

$$\frac{V_o}{V_s} = \frac{V_o}{V_{b2}} \times \frac{V_{b2}}{V_s} \approx \underline{\underline{-101}}$$

d) $R_{is} = R_1 \parallel R_2 \parallel r_{\pi 1} = \underline{\underline{3.52\ k\Omega}}$, $R_o = \dfrac{(R_3 \parallel R_4 \parallel R_{C1}) + r_{\pi 2}}{1+\beta_2} = \underline{\underline{48.9\ \Omega}}$

(e)




dc load line eq.

$12 \simeq V_{CE2} + I_{C2} R_{E2}$

$I_{CQ2} = 4.86\ mA$, $V_{CEQ2} = 4.224\ V$

$R_{ac} = 1.6 \parallel 0.25 = 0.216\ k\Omega$

ac load line eq. $i_{C2} = -\dfrac{1}{0.216} v_{CE2} + b$

$\therefore\ b = I_{CQ2} + \dfrac{V_{CEQ2}}{0.216} = 24.41\ mA$

$$\underline{\underline{i_{C2} = -\frac{v_{CE2}}{0.216} + 24.41}}$$

When $i_{C2} = 0$, $v_{CE2} = 24.41 \times 0.216 = 5.27\ V$

$v_{E2} = 12 - 5.27 = 6.73\ V$. $\dfrac{6.73}{R_{E2}} = 4.21\ mA$

$V_{out} = -4.21 \times R_L = \underline{\underline{-1.05\ V}}$ ($V_{out,min}$)

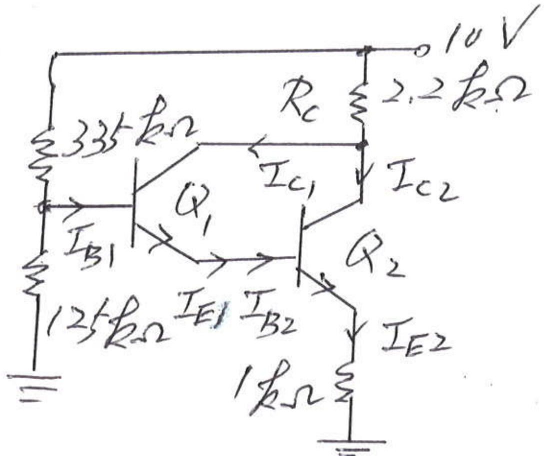
When $v_{CE2} = 0.2\ V$ (Saturation), $v_{E2} = 12 - 0.2 = 11.8\ V$

$i_{C2} = -\dfrac{0.2}{0.216} + 24.41 = 23.48\ mA$, $v_{out} = \left(23.48 - \dfrac{11.8}{R_{E2}}\right) \times R_L$

$\therefore\ \underline{\underline{v_{out} = 4.03\ V\ (V_{out,max})}}$. $\underline{\underline{v_{out,dc} = 0\ V}}$

$\Delta V_{o,peak} = \underline{\underline{1.05\ V}}$

6.78
(a)




Assume $Q_1$ and $Q_2$ are biased in active region.

$V_{BE1} = V_{BE2} = 0.7\,V$

$\beta_1 = \beta_2 = 100$

$$-10 \times \frac{125}{125+335} + I_{B1} \times \frac{125 \times 335}{125+335} + V_{BE1} + V_{BE2} + I_{E2} \times 1 = 0$$

$$-2.72 + 91 I_{B1} + 0.7 + 0.7 + I_{C2} + I_{B2} = 0$$

$$-2.72 + 91 I_{B1} + 1.4 + 101 I_{B2} = 0 \quad \cdots\cdots \text{①}$$

$$I_{B2} = I_{E1} = I_{C1} + I_{B1} = 101 I_{B1} \quad \cdots\cdots \text{②}$$

Combining eqs. ① and ②, we obtain

$$-2.72 + 1.4 + 91 I_{B1} + 101 \times 101 I_{B1} = 0$$

$\therefore$ $I_{B1} = 128\,nA$, $\underline{\underline{I_{C1} = 12.8\,\mu A}}$, $g_{m1} = 0.512\ mA/V$

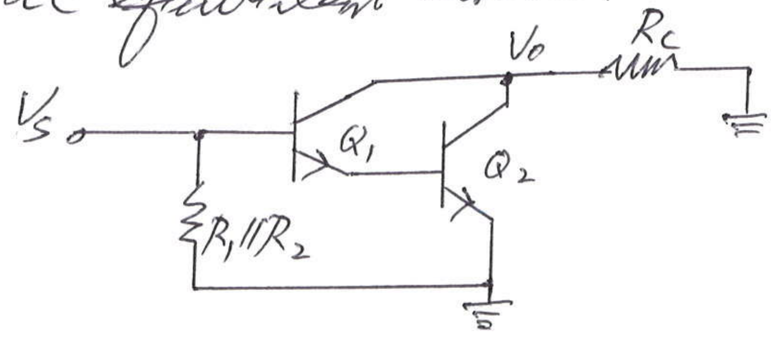
$I_{B2} = 12.9\,\mu A$, $\underline{\underline{I_{C2} = 1.29\,mA}}$, $g_{m2} = 51.6\ mA/V$

$V_{B1} = 0.7 + 0.7 + I_{E2} \times 1 \cong 2.69\,V$, $V_{C1} = V_{C2} = 10 - 2.2(I_{C1} + I_{C2})$

$V_{C1} = V_{C2} = 7.13\,V$ $\qquad V_{BC1} = -4.44\,V$

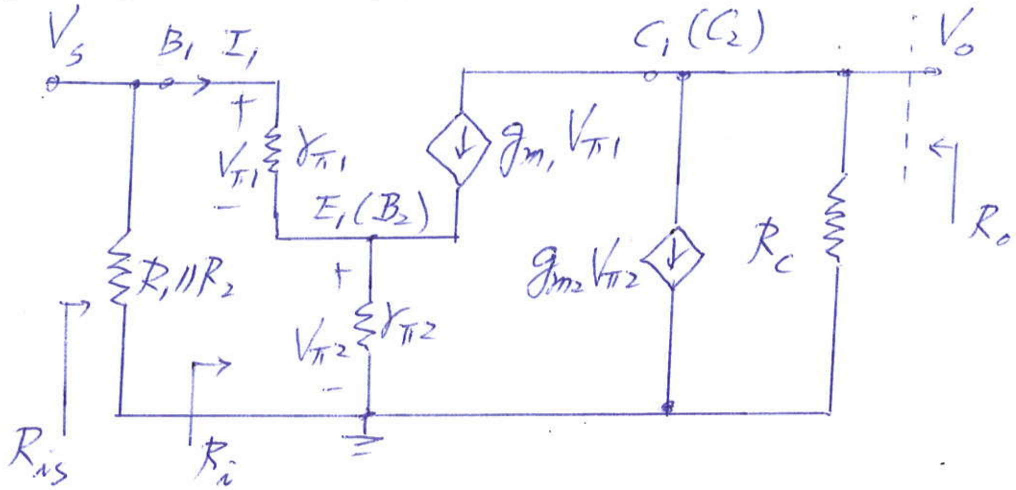
$V_{B2} = 0.7 + I_{E2} \times 1 \simeq 1.99\,V$ $\qquad V_{BC2} = -5.14\,V$

(b) ac equivalent circuit



ac equivalent circuit in detail

6.78 (b)




$r_{\pi 1} = 195.3\ k\Omega$

$r_{\pi 2} = 1.938\ k\Omega$

$$\begin{cases} V_o = -(g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2}) \times R_C = -1.126 V_{\pi 1} - 113.52 V_{\pi 2} \\ V_s = V_{\pi 1} + V_{\pi 2} \\ V_{\pi 2} = \left( \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) \times r_{\pi 2} = 0.01 V_{\pi 1} + 0.99 V_{\pi 1} \end{cases}$$

$\therefore\ V_{\pi 2} \simeq V_{\pi 1}, \quad V_s \simeq 2 V_{\pi 2}$

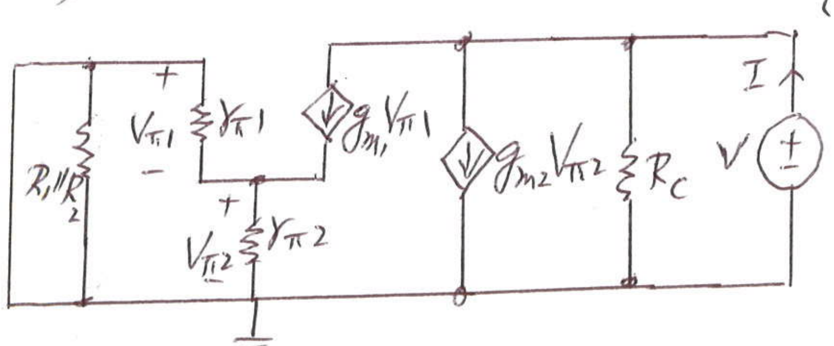
$\therefore\ V_o \approx (-1.126 - 113.52) \times \frac{V_s}{2}$

$$\frac{V_o}{V_s} \simeq -57.3$$

$$R_i = \frac{V_s}{I_1} = \frac{V_s}{V_{\pi 1}/r_{\pi 1}} = \frac{V_s}{V_{\pi 1}} \times r_{\pi 1} \simeq 2 r_{\pi 1} = 330\ k\Omega$$

$$R_{is} = R_1 // R_2 // R_i = 71.3\ k\Omega$$

Output resistance




$$\begin{cases} V_{\pi 1} + V_{\pi 2} = 0 \\ V_{\pi 2} = \left( \frac{V_{\pi 1}}{r_{\pi 1}} + g_m V_{\pi 1} \right) r_{\pi 2} \end{cases}$$

$V_{\pi 1} = V_{\pi 2} = 0$

$$\therefore\ R_o = \frac{V}{I} = R_C = 2.2\ k\Omega$$