Introducing … the Fuzzy Fan!

- Goal: Maintain comfortable temperature and humidity in a residential bathroom.
  - Traditional (Binary) fan
  - Fuzzy fan

- Requirements:
  - Simplify operation
  - Minimize energy consumption
Types of Uncertainty

Stochastic Uncertainty:
- The probability of hitting the target is 0.8

Lexical Uncertainty:
- "tall men", "hot days", or "stable currencies"
- We will probably have a successful business year.
- The experience of Expert A shows that Event E is likely to occur. However, Expert B is convinced this is not true.

Most words and evaluations we use in our daily reasoning are not clearly defined in a mathematical manner. This allows humans to reason on an abstract level!

Philosophical Origins

Georg Cantor
1845 - 1918

Georg Cantor founded (classical) set theory and introduced the concept of infinite numbers with his discovery of cardinal numbers.
What's so "fuzzy" about a Fuzzy Fan?

Philosophical Origins

- Sorites paradox - the paradox of the heap
- Falakros paradox - the baldness paradox
- Theseus’ Ship - after Theseus’ return to Athens from slaying the Minotaur

Charles Dodgson
1832 - 1898

Charles Dodgson was a mathematics lecturer and author of mathematics books who is better known by the pseudonym “Lewis Carroll”; he is known especially for Alice’s Adventures in Wonderland (1865) and Through the Looking Glass (1872).
Philosophical Origins

Charles S. Peirce
1839 - 1914

“Vagueness is no more to be done away with in the world of logic than friction in mechanics.”

Charles S. Peirce, America’s most innovative philosopher, was the first thinker to grapple seriously with *vagueness*.

Philosophical Origins

Bertrand Russell
1872 - 1970

Bertrand Russell is one of the most important logicians of the 20th Century.

“Vagueness, clearly, is a matter of degree.”

Vagueness and precision are features of language, not reality. “All language is vague.”
Philosophical Origins

Albert Einstein
1879 - 1955

“As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.”

An Early Formalism ...

Jan Łukasiewicz
1879 - 1955

Jan Łukasiewicz worked on mathematical logic, developed a three-valued propositional calculus (1920) and worked on many-valued logics; he also introduced the Polish notation.
And Then Finally ...

Lotfi A. Zadeh
(February 4, 1921 - )
Founder of Fuzzy Set Theory
“Fuzzy Sets,” Information and Control, 8:338-353, 1965

Membership with Crisp Sets

Either or ... all or nothing
What's so "fuzzy" about a Fuzzy Fan?

Membership with Fuzzy Sets

Subjective … More-or-less

Membership Functions

Height

my height

Short People

Tall People

0.0

0.5

1.0

0'0"

5'0"

10'0"

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What's so “fuzzy” about a Fuzzy Fan?

Containment in Crisp Sets

Containment in Fuzzy Sets
What's so "fuzzy" about a Fuzzy Fan?

Intersection/Union with Crisp Sets

- Intersection: $A \cap B$
- Union: $A \cup B$

Intersection with Fuzzy Sets

- Height vs. Membership
- Short People
- Tall People

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Union with Fuzzy Sets

Classical, Boolean Logic

George Boole
1815 - 1864

Aristotle
384 B.C. - 322 B.C.

Gottlob Frege
1848 - 1925
**What's so "fuzzy" about a Fuzzy Fan?**

### Fuzzy Logic Operators

#### NOT Operator

<table>
<thead>
<tr>
<th>a</th>
<th>( \neg a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \mu(\neg a) = 1 - \mu(a) \]

#### AND Operator

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a ( \land ) b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \mu(a \land b) = \min(\mu(a), \mu(b)) \]
What’s so ‘fuzzy”about a Fuzzy Fan?

Fuzzy Logic Operators

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a ∨ b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \mu(\tilde{a} \lor \tilde{b}) = \max(\mu(\tilde{a}), \mu(\tilde{b})) \]

Fuzzy Logic Operators

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a → b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \mu(\tilde{a} \rightarrow \tilde{b}) = \max(1 - \mu(\tilde{a}), \mu(\tilde{b})) \]
Probability versus Uncertainty

A
Prob(water) = 0.91

B
Degree(water) = 0.91

Which glass would you drink from?

Probability and Uncertainty

"... a person suffering from hepatitis shows in 60% of all cases a strong fever, in 45% of all cases yellowish colored skin, and in 30% of all cases suffers from nausea ..."

Stochastics and Fuzzy Logic complement each other!
What's so “fuzzy” about a Fuzzy Fan?

**Control Systems**

- **Open-loop Control System**
  - uses actuating device to control the process directly

  ![Open-loop Control System Diagram]

- **Closed-loop Control System**
  - uses a measurement of the output and the feedback of this signal to compare it with the desired output

  ![Closed-loop Control System Diagram]
Why Fuzzy Control?

- Control strategy can be defined at a linguistic level
- a.k.a. “computing with words” …

Fuzzy Controller Architecture

- Fuzzy controller
- Inference mechanism
- Rule base
- Defuzzification
- Input
- Output
What's so ‘fuzzy’ about a Fuzzy Fan?

B. Juliano, May 2003 What’s so “fuzzy” about a Fuzzy Fan? Slide #31

Detail: The Rule Base

Fuzzy controller

Input

Fuzzification

Rule base

Inference mechanism

Defuzzification

Output

B. Juliano, May 2003 What's so "fuzzy" about a Fuzzy Fan? Slide #32

Linguistic Variables

COOL HOT WARM

0 1

40 60 80 100 120

Temperature (°F)

DRY WET MOIST

0 1

20 40 60 80 100

Humidity (%)

LOW HIGH MEDIUM

0 1

250 500 750 1000 1250

Fan Speed (rpm)

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What’s so “fuzzy” about a Fuzzy Fan?

May 23, 2003

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Fuzzy Rules

1. \textbf{IF} \ H \textbf{is} WET \ \textbf{THEN} \ FS \textbf{is} HIGH
2. \textbf{IF} \ T \textbf{is} COOL \ AND \ H \textbf{is} DRY \ \textbf{THEN} \ FS \textbf{is} MED
3. \textbf{IF} \ T \textbf{is} COOL \ AND \ H \textbf{is} MOIST \ \textbf{THEN} \ FS \textbf{is} HIGH
4. \textbf{IF} \ T \textbf{is} WARM \ AND \ H \textbf{is} DRY \ \textbf{THEN} \ FS \textbf{is} LOW
5. \textbf{IF} \ T \textbf{is} WARM \ AND \ H \textbf{is} MOIST \ \textbf{THEN} \ FS \textbf{is} MED
6. \textbf{IF} \ T \textbf{is} HOT \ AND \ H \textbf{is} DRY \ \textbf{THEN} \ FS \textbf{is} MED
7. \textbf{IF} \ T \textbf{is} HOT \ AND \ H \textbf{is} MOIST \ \textbf{THEN} \ FS \textbf{is} HIGH

Representing Fuzzy Rules

- Simple rules (e.g., Rule #1)

\[
\text{IF H is WET THEN FS is HIGH}
\]

\[
(0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 1 \ 1 \ 1)
\]

\[
RI_{AC} = A^{\circ C}
\]
Representing Fuzzy Rules

- Compound rules (e.g., Rule #4)

**IF** T is WARM AND H is DRY **THEN** FS is LOW

\[
\begin{align*}
\text{Temperature (°F)} & : 40, 60, 80, 100, 120 \\
\text{Humidity (%)} & : 20, 40, 60, 80, 100 \\
\text{Fan Speed (rpm)} & : 250, 500, 750, 1000, 1250
\end{align*}
\]

\[
R_{4AB} = A \circ B
\]
What’s so “fuzzy” about a Fuzzy Fan?

Fuzzy Rule Decomposition

IF \( x \) IS \( A \) AND \( y \) IS \( B \) THEN \( z \) IS \( C \)

\( R_{A \rightarrow C} = A^T \cdot C \)

\( C_{A'} = A' \cdot R_{A \rightarrow C} \)

\( C_{B'} = B' \cdot R_{B \rightarrow C} \)

\( C_{A'} \land C_{B'} \)

Detail: Inference Mechanism

Fuzzy controller

Input → Fuzzification → Inference mechanism → Rule base → Defuzzification → Output
What's so “fuzzy” about a Fuzzy Fan?

May 23, 2003

B. Juliano, May 2003 What's so "fuzzy" about a Fuzzy Fan? Slide #41

Fuzzy Inference

IF H is WET THEN FS is HIGH

C' = A'°RI_{AC} = A'°(A'^{T}C)

Fuzzy Inference

IF T is WARM AND H is DRY THEN FS is LOW

C' = (A' ^ B')°R_{AC}^{4} = (A' ^ B')°(A'^{T}C)

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What's so 'fuzzy' about a Fuzzy Fan?

B. Juliano, May 2003

Slide #43

Detail: Fuzzification

- Fuzzy controller
- Fuzzification
- Inference mechanism
- Rule base
- Defuzzification
- Input
- Output

Slide #44

Fuzzification & Fuzzy Inference

IF
H is WET
THEN
FS is HIGH

Clipping Method vs Scaling Method

Input: Humidity is 72%
What’s so “fuzzy” about a Fuzzy Fan?

Fuzzification & Fuzzy Inference

IF T is COOL AND H is MOIST THEN FS is MEDIUM

Input: Temperature is 65°F
Input: Humidity is 72%

Aggregation

IF H is WET THEN FS is HIGH

IF T is COOL AND H is MOIST THEN FS is MEDIUM

Fan Speed (rpm)
What's so 'fuzzy" about a Fuzzy Fan?

May 23, 2003

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Slide #47

Detail: Defuzzification

Fuzzy controller

Input

Fuzzification

Inference mechanism

Rule base

Defuzzification

Output

Max criterion
Mean of maximum (Mom)

\[ y^* = \frac{\sum y \cdot \text{height}(B')} {\sum \text{height}(B')} \]

Center of gravity (COG), or Center of area (COA)

\[ y^* = \frac{\int y \cdot \mu_B(y) \, dy} {\int \mu_B(y) \, dy} \]
Sample Fuzzy Systems

- Blue Circle Cement and SIRA, Denmark
  - fuzzy expert system for cement kiln (Linkman, 1982)
- Hitachi, Ltd. Sendai Subway System, Japan
  - speed and braking controller (Yasunobu & Miyamoto, 1985)
- Unmanned Helicopter Control, Japan
  - radio control by oral instructions (1992+)
- Household Appliances
  - airconditioner (Hitachi)
  - vacuum cleaner (Matsushita)
  - camera (Canon, etc.)
  - microwave oven (Toshiba, Sharp, Sanyo, and Hitachi)
  - VCR (Sanyo and Matsushita)
  - palmtop computer (Sony)
  - washing machine (Matsushita & Hitachi)

Research Areas and Issues

- How to learn/determine if a set of control rules is sufficient, noncontradictory, and noninteractive?
- What mechanism of inference should one use?
- Learning, Adaptive, Self-Organizing Intelligent Systems
  - neural networks
  - genetic algorithms
  - hybrid intelligent systems
- Standardization
  - IEEE Standards P1423 (Defns), P1424 (V.I.), P1440 (Interfaces)
  - ISO Standards
  - IEC Standards IEC 1131-7 (FL components)
Fuzzy Logic Modeling Tools

Fuzzy Logic Toolbox 2.0
The MathWorks Inc.
http://www.mathworks.com

CubiCalc/CubiCalc RTC
HyperLogic Corporation
http://www.hyperlogic.com

FuzzyTECH
INFORM Software
http://www.fuzzytech.com

Fuzzy Logic Control References

- Dr. J's Fuzzy Logic Resources at http://www.ecst.csuchico.edu/~juliano/Fuzzy
- Fuzzy Logic FAQ
- Inform's fuzzyTECH Library at http://www.fuzzytech.com