Name: ____________________

ID No: ____________________

COT 5405: Analysis of Algorithms

Exam III; April 30, 1999

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (a) (15 points) Present an $O(n \log^2 n)$ time algorithm to compute the coefficients of $(x - a_1)(x - a_2) \cdots (x - a_n)$ where $a_1, a_2, \ldots, a_n$ are given scalars.

   (b) **[Bonus Problem]** (10 points) A and B are sets of integers in the range $[0, 5n]$. Define $C_i = \{(x, y) : x \in A, y \in B, \&x + y = i\}$, for $i = 0, 1, \ldots, 10n$. Present an $O(n \log n)$ time algorithm to compute $|C_i|$, for $i = 0, 1, \ldots, 10n$.

2. (18 points) Input are k sets $S_1, S_2, \ldots, S_k$ such that $\sum_{i=1}^{k} |S_i| = n$. The problem is to determine if these sets are pairwise disjoint, i.e., $S_i \cap S_j = \emptyset$, for $i \neq j$. Show that $\Omega(n \log n)$ is a lower bound on the time needed to solve this problem.

3. (16 points) Prove or disprove:

If a problem π is in $\mathcal{NP}$, then π can be solved using a deterministic algorithm that runs in time $O(2^{n^c})$, c being a constant.

4. (16 points) Let $G(V, E)$ be any graph. An *independent set* of G is defined to be any subset V' of V such that each edge of G is incident on at most one vertex in V'. The **Independent Set Problem** (ISP) takes as input a graph $G(V, E)$ and an integer $k \leq |V|$. The problem is to decide if G has an independent set of size k. Is ISP in $\mathcal{P}$? If yes, present a polynomial time algorithm. If not, show that it is $\mathcal{NP}$-complete.

5. (15 points) Input is a graph $G(V, E)$ in adjacency matrix form. The problem is to determine if G is directed or undirected. Present an $O(\log |V|)$ time algorithm for this problem. You can use up to $\frac{|V|^2}{\log |V|}$ CREW PRAM processors.

6. (20 points) The *Linear Congruential Generator* for generating pseudorandom integers uses the recurrence

$$x_i = ax_{i-1} + b (\bmod n)$$

where a, b, and n are given integers, n being a prime. Also, x_1 is given (and is known as the *seed*). Present an $O(\log n)$-time $\frac{n}{\log n}$-processor CREW PRAM algorithm to compute the first n numbers from this sequence.