

Name: _____

ID No: _____

COT 5405: Analysis of Algorithms

Exam I; February 24, 2000

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. **a)** (5 points) Prove or disprove: If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$ then $\frac{f_1(n)}{f_2(n)} = O\left(\frac{g_1(n)}{g_2(n)}\right)$.
- b)** (5 points) Prove or disprove: $n^3 + 8n^2 \log^{10} n = \Theta(n^3)$.
- c)** (5 points) Solve the recurrence relation:

$$T(n) = \begin{cases} 1 & \text{if } n < 16 \\ T(n^{1/4}) + \sqrt{\log n} & \text{if } n \geq 16 \end{cases}$$

2. (20 points) Compute the run time of the following randomized algorithm and express it using the $\tilde{O}()$ notation.

Algorithm Curious

```
{  
  repeat  
    Flip an  $n$ -sided coin; let  $i$  be the outcome;  
    Flip an  $n$ -sided coin; let  $j$  be the outcome;  
    if  $j \leq i$  then quit;  
  forever  
}
```

3. (20 points) SmartStore is an online retail store that wants to develop suitable datastructures to keep information about its customers and transactions. Each transaction can be thought of as a triple (CID, A, t) . This triple corresponds to a customer named CID purchasing goods worth $\$A$ at time t . SmartStore wants to reward customers who buy a lot. Since the amount of memory available is finite, it wants to periodically delete from its datastructure information regarding those customers who have not bought anything in the recent past. In particular, the operations that should be supported are:

ProcessTransaction (CID, A, t) : If customer CID is already in the datastructure, note the fact that this customer has purchased additional items worth $\$A$ at time t . If CID is a new customer, create a new entry for this customer and store the relevant information. This operation should take $O(\log n)$ time to process, where n is the number of active customers.

BestCustomers $()$: return the list of the top 100 active customers in terms of the total amount of purchases made by them until now. This operation should take $O(\log n)$ time to process.

DeleteInactive (t) : delete information regarding all the customers who have not purchased anything at or after time t . This operation should take $O(q \log n)$ time where q is the number of such customers. These customers will be considered inactive.

Present suitable data structure(s) for this problem. Prove your answer. You can use $O(n)$ memory.

4. (15 points) Input is an undirected graph $G(V, E)$ in the form of adjacency lists. Present an $O(|V|^2)$ time algorithm to check if G is complete. Recall that a graph is complete if there is an edge from every node to every other node.

5. (15 points) Two different divide-and-conquer algorithms \mathcal{A} and \mathcal{B} have been designed for solving the problem π . \mathcal{A} partitions π into 5 subproblems each of size $\frac{n}{3}$. Here n is the input size for π . It takes a total of $\Theta(n^2)$ time for the partition and combine steps. \mathcal{B} partitions π into 10 subproblems each of size $\frac{n}{4}$. It takes a total of $\Theta(n^{1.8})$ time for the partition and combine steps. Which algorithm is preferable? Why?

6. (15 points) Let X be a sequence of n keys not necessarily in sorted order. An element x of X is said to be an *extreme element* if $|\{k \in X : k \leq x\}|$ is either $\leq \frac{n}{4}$ or $\geq \frac{3}{4}n$. Present an $O(n)$ time algorithm to output all the extreme elements of X .

7. ****Extra Credit**** (10 points) Input is a sequence X of n random floating point numbers (of arbitrary precision) picked from the interval $(0, 1)$. Present an algorithm to sort X . The expected run time of your algorithm should be $O(n)$.