Before viewing the slide-show and labs, review this page to familiarize yourself with the structure of this tutorial.

Information on Fourier and his Series is presented into three phases:

- **Phase 1: Background and Basics**

  Who is Fourier; What is the series; Why would you care; Some mathematical details; A cool Java applet to let you see the basic concept in action.

- **Phase 2: Applying Fourier To An Engineering Problem**

  Use Fourier to understand a complex real-world problem and some strange electronic equipment designed to combat the problem; An even cooler Java applet to simulate the situation.

- **Phase 3: Proposing A Better Solution Than Compensation**

  The experiments with compensation may be interesting, but the perceptive student will suspect there is a better way. Leveraging lessons learned, a better way is disclosed.
**Background and Basics**

- **Significance of 'Fourier'**
  
  A mention of the person behind the Fourier theories, and their significance to science students and professionals.

- **What is the Fourier Series?**
  
  The basic principle is described. Links to other sites for further reading are listed.

- **Lab 1: Four Common Wave Forms, Decomposed**
  
  See the series in action. The harmonic components and their sum are shown for four common wave forms. The student can control how many harmonics are included in the summation.
Applying Fourier To An Engineering Problem

• An Engineering Problem - Data Transmission

Digital chips can send 'bits' to each other as simple square waves. This doesn't work when the two chips are separated by long distances. Why not? This classic problem showcases the power of Fourier series analysis.

• Lab 2: Square Waves Gone Bad

We will examine the reasons 'bits' won't travel, undistorted, over long lines, as well as 'compensation' techniques for dealing with these problems. Topics covered are:
  o Bandwidth Distortion
  o Attenuation Distortion
  o Attenuation Compensation
  o Delay Distortion
  o Delay Compensation
Proposing A Better Solution Than Compensation

- Leveraging The Benefits of Pure Sinusoidal Waves

By contrasting the expected behavior of square waves and pure sine waves in a long transmission line, the reason for techniques such as 'frequency modulation' become clear.
Only the simple, basic Fourier series, also known as the 'harmonic series', is discussed in this tutorial.

Four common waveforms are used for demonstrations. The series for each of these waves is presented without showing how they were developed. Such analysis is beyond the scope of this tutorial.

More complex Fourier mathematics such as Fourier Transforms or Discreet Fast Fourier Transforms are not discussed here.
Fourier: The Man - The Myth - The Math

A French mathematician, Joseph Fourier (1786-1830) developed a powerful method for analyzing 'periodic waves'. This, and other related developments, have become fundamental building-blocks for understanding and predicting wave-related phenomena in a variety of scientific fields - communications, chemistry, acoustics, and countless others.

Fourier's work has become an integral part of every science curriculum, appearing in basic courses such as Calculus and Physics, and permeating specialties such as Electrical Engineering. Students and practicing scientific professionals will often encounter the Fourier name, for example "Fourier transforms" and "discrete fast Fourier transforms (DFFT)". Chemists may find themselves working with a "Fourier transform infrared (FTIR) spectrometer system" to identify chemical substances.

A student of Natural Sciences who truly understands the work of Fourier is at a great advantage. The purpose of this on-line tutorial is to illustrate the most basic Fourier principle, the "Fourier Series". The principle is explained, of course, but the tutorial goes on to show how it can be used to understand and 'solve' some real problems encountered in computer networks.
The 'Fourier Series' Concept, simply stated...

Any periodic waveform can be represented as a combination of simple harmonics.

Click on the links below to examine each of these terms in more detail:

Periodic Waves
Combination of Waves
Simple Harmonics
Mathematical Details
These 3 waves are 'periodic'.

The same pattern repeats every $t=T$ interval.

All have the same 'frequency'.

This wave is **not** 'periodic'.

$T$ = 'period'

$f$ = 'frequency' = $1/T$

$\omega$ = 'angular frequency' = $2\pi f$ = $2\pi/T$
The upper wave is the 'combination' or 'sum' of the two lower waves.
This wave is a 'simple' wave or 'simple harmonic'
This means it is a pure sinusoidal wave:

\[ y = \sin(\omega t) \]  or  \[ y = \cos(\omega t) \]

Any waves other than \( \sin(\omega t) \) or \( \cos(\omega t) \) are *NOT* simple. Let's call them 'complex'.

**We can re-state the Fourier Series:**

Any periodic, complex wave can be visually represented, mathematically expressed, or physically synthesized by adding the right combination of pure sine or cosine waves together.
For the sake of completeness, the formal mathematical version of the Fourier Series is presented here. In the pages ahead, the derived series for sample waveforms will be presented without derivation, and without proof.

The complete, generalized Fourier Series is expressed as:

\[ v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \]

The terms 'a' and 'b' are known as the *Fourier coefficients*, and for a particular waveform, we can derive them from the following set of integrals:

\[ a_0 = \frac{1}{T} \int_{0}^{T} v(t) dt \]

\[ a_n = \frac{2}{T} \int_{0}^{T} v(t) \cos(n\omega_0 t) dt \]

\[ b_n = \frac{2}{T} \int_{0}^{T} v(t) \sin(n\omega_0 t) dt \]
Why would anyone possibly care about replacing a 'complex' wave with a bunch of sine waves?

The answer is that sine waves are well-understood.

- Classical Calculus and Algebra are loaded with proven formulas for manipulating sinusoidal functions.
- The physical sciences can often predict the behavior of sinusoidal waves (electromagnetic, light, sound, or mechanical vibrations) in media.

The Fourier Series enables a "divide and conquer" approach to solving problems that deal with complex waves.

Math Example: Square Waves
Physical Example: Cheap Speakers
It's easy to write a mathematical function to describe the sine wave: \( y = \sin(\omega t) \). If you have a need to manipulate this function, such as find its first derivative, classical Calculus and Algebra have been developed to guide you.

A mathematical formula for any other wave can be a real mess; even something as common as the 'square wave' used to send data between digital circuits is difficult to describe with a "\( y = f(t) \)" type of formula. Even if you manage to come up with one, classical Calculus and Algebra cannot be used to work with it.

The Fourier Series is the solution to this dilemma! The square wave is a 'periodic waveform', and you can represent it mathematically as a summation of pure cosine waves as follows:

\[
y(\omega t) = a_1 \cos(\omega t) - a_3 \cos(3\omega t) + a_5 \cos(5\omega t) - a_7 \cos(7\omega t) + \ldots
\]

- \( a_1 \cos(\omega t) = \) 'fundamental'
- \( -a_3 \cos(3\omega t) = \) 'third harmonic'
- \( a_5 \cos(5\omega t) = \) 'fifth harmonic'

The '...' indicates this is an 'infinite series'. Fortunately, Calculus and Algebra are loaded with techniques for dealing with these, so our Fourier Series formula for a square wave is still quite useable.

See Mathematical Details for how the sequence of "term coefficients" \( (a_1, a_3, a_5, \text{ etc.}) \) was ever figured out. That's beyond the scope of this tutorial. However, it is important to note that each periodic waveform will have it's own unique set of term coefficients. Characteristics that you should notice for each wave include:

- Odd-only, even-only or odd/even harmonics included
- alternating '+' and '-', all + or all -
- \( a_n \) is usually a function of "The Fourier Constant", \( \pi \), and \( 1/n \) or \( 1/n^2 \)
Why do "little radios" or any sound reproduction/amplification system with "cheap speakers" sound so bad? Why spend lots of money on "high quality" speakers?

The reason can be understood with the Fourier Series.

- Input a 'pure sinusoid' into the "high quality" system, and the output will also be a 'pure sinusoid'.

- Input the same 'pure sinusoid' into the cheap system, and the output will **not** be a pure sinusoid - it has been misshapen or 'distorted'.

From our definition of simple harmonics, we know that the distorted wave **must** be made up from several sinusoidal waves - yet... we only put a single wave in?

It is these 'harmonics' that were **added** by the cheap system that causes the sound to be unpleasant.

'Perfect reproduction' of an input signal is a very difficult thing to do, and usually requires expensive audio components, especially speakers. The cheaper the speaker, the more it will change the shape of the input signal - the more it will 'distort' it - the more it will add harmonics.
In this first laboratory exercise, we will see the basics of the Fourier Series in action. We will use it to synthesize five common wave forms.

- Starting The Applet
- Square Waves
- Sawtooth Waves
- Full-Recitifed Sine Waves
- Triangle Waves
- Sine Waves
- Lab 1: Conclusions
If viewing as HTML, click here to [Start The Applet](#) in a separate window. It runs in a separate window so that you can shrink the Browser frame to as small as needed, and see both the applet and the experiment steps at the same time.

If viewing as PDF, click the bookmark "Demo of 5 ..." in the navigation pane. You must click the browser "Back" key to return to the tutorial.

The applet is simple to use:

- **Control Panel**
  - select waveform with radio buttons
  - add/remove harmonics with spin buttons
  - digit tells count of harmonics, not n-order
  - maximum of 30 harmonics allowed

- **Summation Panel**
  - 'perfect' wave is in light gray
  - 'synthesized' wave is in blue

- **Harmonics Panel**
  - All harmonics graphed on top of each other
<table>
<thead>
<tr>
<th>CSCI-399I Master's Project</th>
<th>Fall 2001</th>
<th>Jerry Loyd</th>
</tr>
</thead>
</table>

Your browser does not support Java applets
The Series for square waves is:

\[ y(\omega t) = \frac{4}{\pi} \left\{ \cos(\omega t) - \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\cos(5\omega t) - \frac{1}{7}\cos(7\omega t) + \ldots \right\} \]

- **Square Wave Series Patterns**
  - only odd harmonics
  - only cosine terms, no sine terms
  - alternating + / - on coefficients
  - coefficients are function of \(\frac{1}{n}\)

- **Play Around**
  - Click the radio button labeled 'Square'.
  - Click the down-arrow to add harmonics
  - Watch the blue wave in the Summation Panel change shape to be more and more like the gray 'target wave'.

- **Things To Notice**
  - The blue wave looks kind of ragged... The Fourier Series is an 'infinite' series, yet this applet only allows adding up to 30 harmonics
  - After about 15 harmonics, adding more has little effect. Theoretically, if the applet allowed an infinite number, the wave would eventually be a perfect square wave. Chances are, round-off error and other real-life restrictions will prevent this 'perfect' wave from ever being synthesized.

- **Find The Fewest Harmonics Needed...**
  - Remove harmonics until, in your opinion, the square wave is so crude it's just not a square wave any more. Add a few, remove a few, and decide on the fewest harmonics needed. Of course, criteria for "minimally acceptable" changes depending on what one wants to do with a square wave, but generally, the answer is ...
    - between 3-to-5 harmonics are needed
    - or... at least 5-to-7 times the frequency of the fundamental is needed
    - A guy named Nyquist says... \(6\omega\) is needed.
The Series for sawtooth waves is:

\[ y(\omega t) = \frac{1}{\pi} \{ -\sin(\omega t) - \frac{1}{2}\sin(2\omega t) - \frac{1}{3}\sin(3\omega t) - \frac{1}{4}\sin(4\omega t) + \ldots \} \]

● **Sawtooth Wave Series Patterns**
  - odd and even harmonics
  - only sine terms
  - all '-' multiplier on coefficients
  - coefficients are function of 1/n

● **Play Around**
  - Click the radio button labeled 'Sawtooth'.
  - Click the down-arrow to add harmonics
  - Watch the blue wave in the Summation Panel change shape to be more and more like the gray 'target wave'.

● **Things To Notice**
  - The blue wave looks kind of ragged here too...
  - Like the square wave, adding more than 15 harmonics has little effect.

● **Find The Fewest Harmonics Needed...**
  - Generally, the answer is ...
    - **about 6 harmonics are needed**
    - **or, about 6 times the frequency of the fundamental is needed**
    - **Nyquist says about 6*\(\omega\) is the right answer, same as for square wave.**
The full-rectified wave is one used in direct-current power supplies; it enables converting "ac" power into "dc".

The Series for the full-rectified sine wave is:

\[ y(\omega t) = \frac{1}{\pi} + \frac{1}{4} \left\{ - (1 - \cos(\omega t)) - \frac{1}{2}(1 - \cos(2\omega t)) - \frac{1}{3}(1 - \cos(3\omega t)) - \ldots \right\} \]

- **Full-Rectified Wave Series Patterns**
  - odd and even harmonics
  - funny-looking "1-cos" terms
  - all '-' multiplier on coefficients
  - coefficients are function of 1/n

- **Play Around**
  - Click the radio button labeled 'Full Rectified'.
  - Click the down-arrow to add harmonics
  - Watch the blue wave in the Summation Panel change shape to be more and more like the gray 'target wave'.

- **Things To Notice**
  - The blue wave looks reasonably smooth
  - Adding more than 4 harmonics has little effect

- **Find The Fewest Harmonics Needed...**
  - Generally, the answer is ...
    - only 2 harmonics are needed
    - or, about 3 times the frequency of the fundamental is needed
The Series for the Triangle wave is:

\[ y(\omega t) = \left(\frac{4}{\pi^2}\right) \{ \cos(\omega t) + \frac{1}{9} \cos(3\omega t) + \frac{1}{25} \cos(5\omega t) + \frac{1}{49} \cos(7\omega t) + \ldots \} \]

- **Sawtooth Wave Series Patterns**
  - odd-only harmonics
  - all '+' multiplier on coefficients
  - coefficients are function of \( \frac{1}{n^2} \) - notice that even by the 3rd term, not much is being added to the total.

- **Play Around**
  - Click the radio button labeled 'Triangle'.
  - Click the down-arrow to add harmonics
  - Watch the blue wave in the Summation Panel change shape to be more and more like the gray 'target wave'.

- **Things To Notice**
  - The blue wave looks reasonably smooth
  - Adding more than 2 harmonics has little effect

- **Find The Fewest Harmonics Needed...**
  - Generally, the answer is ...
    - *only 1 harmonic is needed*
    - *or, about 3 times the frequency of the fundamental is needed*
This trivial example is included in the demo to hammer home the point that the 'simple' sine wave is radically different than the other four 'complex' waves.

The Series for sine waves is:

\[ y(\omega t) = \sin(\omega t) \]

- **Sine Wave Series Patterns**
  - there are *no* harmonics
  - only a *single* term, not an infinite series

- **Play Around**
  - Click the radio button labeled 'Sine'.
  - Click the down-arrow to add harmonics - it's a waste of time!

- **Things To Notice**
  - The blue wave is *always* a perfect match of the 'target wave', in fact you can't see the target wave because it is perfectly covered

- **Find The Fewest Harmonics Needed...**
  - The answer is ...
    - **ZERO** harmonics are needed
    - or, only the *frequency of the fundamental* is needed

This fact...

**transmitting a sine wave of frequency \( \omega \) only requires 'bandwidth' of \( \omega \)**

will be examined in greater depth in the following labs. Other benefits of this will emerge. We will use this knowledge to understand why 'amplitude modulation' (AM) is a lousy way to transmit data.
Periodic waves really can be broken down into a 'series' of sine and cosine waves - even funny-looking waves like full-rectified.

- Even though the series is 'infinite', the series of sine and cosine waves will be easier to mathematically manipulate than the complex wave we started with.
- In most cases, the behavior of pure sine and cosine waves in nature is well-understood and very predictable. By knowing the series for a complex wave, the behavior of a complex wave in nature can then be understood and predicted by 'summing the behaviors' of all the harmonics.

We can synthesize a given periodic wave

- We need to know the series for that wave form
- We cannot ever 'perfectly' synthesize a wave, because the Fourier Series is an 'infinite series'
- Nonetheless, sometimes even 1 or 2 harmonics is all that is needed to create a 'reasonable' version of the desired wave

Each wave form has a unique set of harmonics

- The formulas are visually different from each other
- The overlaid pattern of harmonics is visually unique and easily identified
- A 'fingerprint' or small set of identifying characteristics could be created for wave forms. Actually, it would not be that difficult to build an analog circuit that could identify a number of 'known' periodic waves. This is the fundamental principal behind all the specialized equipment with 'Fourier' in their names that can identify a variety of things by their harmonic fingerprints.

The pure sine wave is very different from any of the complex waves

- It has only one component - itself!
Digital chips can send 'bits' to each other as 1's and 0's, essentially square waves. This doesn't work when the two chips are separated by long distances. Why not? This classic problem showcases the power of Fourier Series analysis.

- **So... Who cares?**

  Only a few years ago, most people wouldn't care about this topic, aside from some specialized Electrical Engineers. These days, nearly everyone is obsessed with this problem as the demand and desire for faster and faster "Internet access" or faster and faster "browsing" overcomes us all!

- **What is the source of the problem?**

  The wiring and cables needed to transmit data have many real-world limitations. Jump to the series of slides below to explore this in detail

  - The Perfect Wire
  - Real-World Cables
  - Real-World Cables

  These limitations cause 'distortion' to all signals they transmit.

In the following lab, we will use the Fourier Series to learn about 3 kinds of distortion, how they can affect transmitted signals, and even some tools for combating 2 forms of distortion.
The new Engineer tasked with the design and layout of a real digital circuit board is often surprised by the plethora of 'analog' problems that are encountered. As signal frequencies get higher and higher, transmitting a clean digital signal even a few inches across the printed-circuit (PC) board can become a challenge.

When students (and Engineers) draw 'connections' or 'wires' between one chip and another, the assumption is that these wires are "perfect". A perfect wire has the following characteristics:

- The signal coming out of the end of the wire is an exact copy of the signal that went in
- There is no time-delay between the two ends of the wire

Of course, there is no such thing as a perfect wire like this. All real-world wires will distort and delay all signals that pass through them.
When a 'wire' gets longer, and must connect one digital device to another, we no longer call it a 'wire', but rather a 'cable'.

There's no such thing as a 'perfect' cable either. In fact, whatever problems one might encounter with 'wires', they will see ten-fold with cables. This is because real-world phenomena such as 'inductance' (L), 'resistance' (R) and 'capacitance' (C) exist in every wire and cable, to varying degrees. The generally-accepted model of a 'cable' is shown below:

![Cable Diagram](image)

For excellent, in-depth explanations of cable characteristics, check out the following web pages:

- Belden Cable Technical Technical Papers
- cabletesting.com Measurement Definitions
The most important things to remember about cables at this time are:

- No cable is perfect; all cause distortion, all delay signals
- As desired signal frequency ($\omega$) increases, problems get worse
- Many technologies have been introduced, in search of the perfect cable
  - twisted pair
  - coax
  - fiber optic
  - beyond cables... microwave, etc.
- Typically, as the capability of a cable gets "better" (i.e., able to transmit higher frequency over longer distances...) the costs also get higher - much higher.
The focus of this lab is to use the Fourier Series concept to see just how cables distort signals. We do this by examining the effect the cable can have on each of the harmonics that make up an input signal, then adding them all up to see the composite wave that 'comes out the other end' of the cable.

Additionally, once we know how cables can introduce problems, we can examine some techniques for counteracting the effects of the cable, also known as 'compensation techniques'.

The same five types of wave forms that were used in Lab 1 are used here. The astute student will notice that some waves are less-easily messed up than others. This observation leads us to the final topic in the tutorial.

Jump to the slides below to carry out Lab 2:

Start the Fourier Workbench
Bandwidth Distortion
Bandwidth Distortion
Bandwidth Distortion
Attenuation Distortion
Attenuation Compensation
Amplitude Equilization
Delay Distortion
Delay Compensation
Lab 2: Conclusions
Review this slide to learn how to use the 'Fourier Workbench' Applet.

If viewing as HTML, click here to start the Fourier Workbench, in new Window.

Click 'Next' above to begin the laboratory exercises.

If viewing as PDF, click the "Fourier Workbench" bookmark in the navigation pane. You must click the browser's "Back" button to return to the tutorial.
The first and simplest form of distortion we will examine is 'bandwidth' distortion.

Bandwidth distortion exists because all physical cables have some 'cutoff frequency', i.e., harmonics with frequencies greater than this cutoff value will not transmit through the cable.

Different types of cables have different cutoff frequencies. The actual cutoff value is never exact, however, the following three examples are so radically different from each other, you'll get the idea:

<table>
<thead>
<tr>
<th>Cable type</th>
<th>Cutoff Frequency</th>
<th>Benefits/Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>'twisted pair' (like telephone cable)</td>
<td>1 Million bits per second (Mbps) for distances less than 100 meters</td>
<td>cheap... but slow</td>
</tr>
<tr>
<td>'Coaxial' (like cable TV)</td>
<td>10 Mbps, over several hundreds of meters</td>
<td>sturdy, cheap simple connectors</td>
</tr>
<tr>
<td>'fiber optic'</td>
<td>100's of Mbps over miles</td>
<td>fast... but expensive and requires complex connection tools</td>
</tr>
</tbody>
</table>

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The Fourier Series shows us why the bandwidth limitations of a cable make it impossible to ever transmit a perfect reproduction of any signal from one end to the other:

- The Fourier Series is an "infinite series"
- Each successive term of the Series increases in frequency
- Eventually... the terms have frequencies that are greater than the cutoff frequency of the cable - they won't go through the cable

Therefore, only a finite number of the harmonics will be transmitted - the received signal will be missing many terms, and therefore cannot look exactly like the source signal. It will be distorted.

This is not a 'theoretical problem', but rather a very real one. No matter how good cables get, the demand for data and the cost of cables will encourage Communications Engineers to always push the limits of whatever cable is available.
You have already seen the effects of Bandwidth Distortion in Lab 1, when you added and subtracted harmonics to find the fewest harmonics needed to make a 'reasonable' wave for each type.

To repeat that exercise with the Fourier Workbench, do the following:
- Make sure all the 'distortion' and 'compensation' check-boxes are un-checked
- It doesn't matter which 'Distortion Model' is selected... this is unused when 'distortion' is unchecked
- Use the down-arrow under the Harmonics label to add harmonics... You will see individual strips added, one beneath the other, in the Harmonics Panel.
- Use the up-arrow to remove harmonic strips.

Watch the 'composite output wave' in the top wave strip, to see the wave that would come out of a cable capable of transmitting only the selected number of harmonics.

Play around with all waveforms. Assuming the answers given earlier for the least number of harmonics required for each waveform, the following table can be constructed for the 3 cable types we have discussed:

<table>
<thead>
<tr>
<th>Cable Type</th>
<th>Waveform</th>
<th>Highest Frequency Of A 'Reasonable' Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twisted Pair</td>
<td>Square, Sawtooth</td>
<td>167 KHz</td>
</tr>
<tr>
<td>~ 1 Mbps cutoff</td>
<td>Triangle, Full Rectified</td>
<td>333 KHz</td>
</tr>
<tr>
<td></td>
<td>Sine</td>
<td>1 MHz</td>
</tr>
<tr>
<td>Coax</td>
<td>Square, Sawtooth</td>
<td>1.67 MHz</td>
</tr>
<tr>
<td>~ 10 Mbps cutoff</td>
<td>Triangle, Full Rectified</td>
<td>3.33 MHz</td>
</tr>
<tr>
<td></td>
<td>Sine</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Fiber Optic</td>
<td>Square, Sawtooth</td>
<td>83 MHz</td>
</tr>
<tr>
<td>~ 500 Mbps cutoff</td>
<td>Triangle, Full Rectified</td>
<td>166 MHz</td>
</tr>
<tr>
<td></td>
<td>Sine</td>
<td>500 MHz</td>
</tr>
</tbody>
</table>
In reality, cables do not have an abrupt 'cutoff frequency'. Here's what really happens:

"As a signal propagates along a transmission medium, its amplitude decreases. This is known as signal attenuation. Signal attenuation increases as a function of frequency."

The Fourier Series can help us understand why this distorts waves as they travel through a cable.

Make the following settings on the Fourier Workbench:
- Wave Shape: Square
- Distortion Model: Linear
- Harmonics: at least 12
- Attenuation Distortion: checked

You will now see two sine waves in every harmonics strip; the gray wave is the original un-attenuated wave, and the blue wave is the attenuated wave.

Scroll through the harmonics strips; notice that as the frequency gets higher and higher, the blue wave is smaller and smaller. By about the (-15 ω) strip, the blue wave is just a line. Higher frequency strips no longer add anything to the composite wave.

Play around with all the wave types, and both Linear and Exponential settings. The 'Exponential Distortion Model' is very extreme, and simulates trying to send a signal down a cable that is way too close to the cutoff frequency of that cable.

You should be getting the idea that some kinds of waves are better choices than others, when it comes to shoving lots of data down a long cable. With exponential distortion selected, the square wave is hopeless, the sine wave is virtually unaffected.
Recall the statement:

"As a signal propagates along a transmission medium, its amplitude decreases."

- This means that cables can only be a limited length; go beyond that length and there will be no signal left.
- Yes... every type of cable will be able to transmit differing frequencies different lengths, but essentially, all cables would only be useful up to a given length.
- In all cases, that length would not be adequate to create the modern-day Internet.

The solution is: 'amplifiers' - also called 'repeaters' - are inserted at intervals along the cable to restore the received signal to its original level.

An amplifier like your stereo amplifier, won't work for this. Why not? Recall the statement:

"Signal attenuation increases as a function of frequency."

A music amplifier tries to amplify all frequencies the same amount. In the next slide, we will examine a better type of amplifier, the Equalizer.

For technical information on the subject of equalizers, use your favorite Internet search engine and the keyword "equalizers". Most information will be about 'audio equilizers' but the concept is the same as for the one we will study next.

Example:  http://www.cardiscountstereos.com/Equalizers.htm
We are going to start with the worst-case settings for a wave - a square wave with exponential attenuation. Make the following settings on the Fourier Workbench:

- Wave Shape: Square
- Distortion Model: Exponential
- Harmonics: at least 4
- Attenuation Distortion: checked
- **Attenuation Compensation: checked**

When you check 'Attenuation Compensation', you will see the Gain sliders for each of the harmonics strips become enabled.

These sliders comprise the world's most ideal equalizer:

- There is a slider for *exactly* the frequency needed by the strip
- The slider is calibrated so that at full adjustment, it just happens to provide exactly the amount of 'gain' needed to restore that strip to its original value (i.e., the blue wave will cover the gray wave)

Play with the sliders for each strip, and see how you are able to change the un-compensated wave back into a 'reasonable' square wave. Notice the 'gain' readings for each slider, and that higher frequencies are amplified much more than the lower frequencies.

**Important Things To Notice:**

- With exponential distortion, adding harmonics beyond '4' is a waste of time.
- Compensation with the gain sliders is very effective
- It is much cheaper to insert an equalizer in a long cable than to replace the cable with one that has 'more bandwidth'
- In fact, a 'better cable' may not even exist, so 'equalizers' would be the only realistic solution

Experiment with other settings - other wave shapes, both linear and exponential distortion, and fewer and more harmonics.

Once again, a very important conclusion here is that the more complex waves - square and saw tooth - need a lot of compensation to be restored, whereas the simple sine wave would do just fine with a simple 'amplifier'.

Remember - the equalizer modeled here is 'perfect' or 'ideal'. A real-world equalizer would not have exactly one slider for exactly the frequencies needed, and each slider would provide the same gain at full adjustment.
The rate of propagation of a sinusoidal signal along a transmission line varies with the frequency of the signal.

Visually, what this means is that all the harmonic strips start out all 'lined up', but as they travel through the cable, they become misaligned.

To see the effects of delay distortion on each of the different wave forms, make the following settings on the Fourier Workbench:

- Wave Shape: Square
- Distortion Model: Linear
- Harmonics: at least 12
- Attenuation Distortion: un-checked
- Delay Distortion: checked

Look at all 5 waveforms. The following table summarizes the effect that delay distortion has on each type:

<table>
<thead>
<tr>
<th>Wave Shape</th>
<th>Linear Delays</th>
<th>Exponential Delays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>Unrecognizable</td>
<td>Unrecognizable</td>
</tr>
<tr>
<td>Saw Tooth</td>
<td>Unrecognizable</td>
<td>Unrecognizable</td>
</tr>
<tr>
<td>Full-Rectified</td>
<td>Recognizable</td>
<td>Looks like saw tooth</td>
</tr>
<tr>
<td>Triangle</td>
<td>Reasonable</td>
<td>Looks like sine wave</td>
</tr>
<tr>
<td>Sine</td>
<td>Mostly Unaffected</td>
<td>Mostly Unaffected</td>
</tr>
</tbody>
</table>

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Fall 2001  
Jerry Loyd
The Fourier Workbench includes 'Delay Compensation'.

- To experiment with delay compensation, go to the Fourier Workbench and click the 'Delay Compensation' check box.
- The Delay sliders are then enabled. These are very similar to the Gain sliders, and effectively these comprise the worlds most ideal 'Delay Equalizer'.
- These allow you to individually slide the harmonic strips back and forth; ultimately, you can use them to realign all the strips so that the received waveform has no delay distortion.

There is no organized exercise here - just play around with the sliders and various settings of wave shape and distortion models.

Delay Equalizers actually exist. Search for "delay equalizer" with your favorite Internet search engine. One example product can be found at: http://www.tellabs.com/products/analog/#amp

**Important Things To Notice:**

- Delay distortion and compensation has the most impact on the complex wave forms of all 3 types of distortion.
- The simple sine wave is virtually unaffected by delay.
The Fourier Series can help us understand a very complex natural observation: When an electronic signal is transmitted down a cable, the signal that is received on the other end can never be a perfect copy of the source signal.

At least 3 forms of 'distortion' cause the received signal to differ from the source:

- Bandwidth - all cables have a limit for transmitting frequencies. Regardless of what this limit is, only some of the components that make up the infinite Fourier Series will be transmitted, all others will be lost.

- Attenuation - each harmonic of the source is attenuated a different amount; the higher the frequency, the more it's attenuated. Essentially, the 'Fourier coefficients' of the source signal are changed by the time they get to the destination.

- Delay - faster harmonics travel down the cable at higher velocities. When the harmonics arrive at the destination, they are misaligned. Delay distortion is the most devastating form of distortion.

Compensating 'repeaters' are needed to transmit data more than a few miles on even the best real-world cable:

- Attenuation would cause all signals to die out, eventually. Repeaters placed at regular intervals along the cable counteract this problem.

- Amplitude Equalizers, which amplify different frequencies a different amount, not only boost the signal, they counteract the problems caused by the increasing attenuation of the higher frequencies.

- Delay Equalizers can realign the harmonics.

Some waveforms are more affected by the 3 forms of distortion than others

- Square waves are easily trashed on their journey down a cable

- Sine waves seem to be virtually unaffected by these 3 forms of attenuation. If the sine wave is within the bandwidth limitations of the cable, and simple repeaters are placed at the right intervals, the sine wave will arrive at the destination with the least amount change from the source signal.
After working with the Fourier Workbench in Lab 2, the student should be able to identify with these two statements:

- Square waves don't travel down cables very well.

- If there were some way to transmit data using only Sine waves, it seems it would have some huge benefits over using square waves or any other complex wave form:
  - Maximum use of the bandwidth of any given cable
  - Impervious to attenuation and delay distortion

Indeed, advanced techniques for transmitting data using 'sine waves' certainly has been developed. Consider that the standard telephone line (a.k.a. public switched telephone network line, or PSTN line) is only designed with a 3000 Hz bandwidth. Using brute-force square waves, only one-sixth that frequency, 500 bits per second, could be transmitted under ideal conditions. How is it that contemporary modems boast transmission speeds of 56K bps or higher?

These advanced data transmission techniques all fall into the category of "modulation". The binary data is 'modulated' into sine waves before being transmitted, and when it is taken off the cable, it must be 'demodulated' back to binary data. The common term for this hardware that performs modulation/demodulation is "modem".

Jump to the series of slides below to look more into the topic of Modulation:

- Amplitude-Shift Keying
- Modulation and Beyond
- Modulation and Beyond
The fancy name at the top of this page is the simplest form of using "only pure sine waves" to transmit data down a cable:

- Transmit a fixed frequency or 'tone' to represent a '1' bit
- Transmit nothing, or 'silence' to represent a '0' bit

The 'amplitude' of the tone is modulated (actually multiplied) by the data. Hence the name amplitude-shift keying.

This is only 'better' than a square wave, if the 'demodulator' can detect the tone quickly. If it takes exactly 6 sine-wave cycles to detect the tone, then the fastest data rate would still only be the same as if you used a square wave.

Real-world tone detectors can actually work in much less time - they only need a single sine-wave cycle. Hence, even this simplest form of modulation gains us the huge benefits we envisioned: higher data rates with resistance to attenuation and delay distortion.

Quite a bit of mathematics has been developed to examine this method of modulation (and all the others, too) to prove things such as the theoretical limits of data rates. Most of this math makes use of the Fourier Series. The study of this math is beyond the scope of this tutorial, and left to the adventurous student.
There always has been, and always will be, a huge demand for faster and faster methods of long-distance communication. The world has an insatiable appetite for 'bandwidth'.

No matter what the current technology might be, there will be scores of clever people - mathematicians, electrical engineers, physicists, computer scientists and so forth - researching ways to push the limits of data transmission further and further. Because there are usually lucrative financial rewards for those who succeed, there are also always scores of smart business people supplying the money for these researchers to continue their work.

Keeping this in mind, a historical study of the advancement of data communications shows continuous advancement along several technical fronts:

- **Exploitation of existing cables and cable networks**

  The biggest opportunity, and the biggest challenge, is to make use of all the cabling that is already in place. The costs of this existing infrastructure is so high, simply tearing it all out for something better is out of the question.

  The most marvelous example of this is seen in the commonplace modem: 20 years ago '300 baud' was considered fast, now '56K baud' is considered typical. This has occurred without whole scale replacement of the telephone system.

  Progress has come from two major branches of technology:

  - **Modulation Techniques**: research in the physical sciences have revealed parameters beyond simple 'voltage amplitude' and 'frequency' that can be sent down a cable. 'Phase' - the instantaneous relationship between current and voltage - is a big one. By controlling and measuring combinations of parameters and their relationships to each other, we can now send multiple bits of data in parallel, a huge boost to data rates.

  - **Error Detection and Correction Protocols**: the real world is tough on data traveling through cables. We can't wait for the perfect cable or the perfectly 'clean' electrical environment. Computer Scientists have developed, refined and optimized many protocols to detect when an error has occurred, and many ways to correct them. The now well-known TCP/IP protocol is an excellent example of this. These advanced protocols are getting more aggressive at testing the environment and seeking the highest data rate possible. Earlier protocols tended to always assume "worst case" conditions, so *everyone* had to run slow even if it wasn't really necessary.
- **Development of cable technologies**
  
  Cable designs, cable construction, and cable installation methods keep getting better and better. 'Fiber Optic' is way better than two copper wires. Coax cables have gotten 'cleaner', more consistent, and cheaper.

- **Wireless communications**

  The growing appetite for 'mobile communications' is pushing the capabilities of wireless communications to higher and higher levels. It is also pushing the costs further and further downward. Everything we've learned from cables - modulation techniques, protocols, and so forth - can be used as a foundation for pushing wireless to new limits.
This tutorial, with embedded Java applets, has attempted to communicate the following:

- Explanation of the Fourier Series concept
- Why the Series is so important
- Common waveforms really can be synthesized with the Series
- An interesting, real-world application of the Series
- Using the Lab experience to understand Modulation
The search engine is not implemented, as the FrontPage search components depend on the use of Microsoft Web servers. This page is placed here so that the 'Search' item will appear in the Banner of every page.

If viewing this tutorial in PDF, use the "Find" icon (binoculars) at the top of the frame to find words or phrases throughout the tutorial.
Under construction

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Many thanks, and much credit, must be given to Mr. Fred Halsall for his inspirational lesson on the transmission of electrical signals, and the enlightening use of the Fourier Series, in his textbook: