COT 5405: Analysis of Algorithms

Exam III; April 30, 1999

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (a) (15 points) Present an \(O(n \log^2 n)\) time algorithm to compute the coefficients of \((x - a_1)(x - a_2) \cdots (x - a_n)\) where \(a_1, a_2, \ldots, a_n\) are given scalars.

(b) **[Bonus Problem]** (10 points) \(A\) and \(B\) are sets of integers in the range \([0, 5n]\). Define \(C_i = \{(x, y) : x \in A, y \in B, & x + y = i\}\), for \(i = 0, 1, \ldots, 10n\). Present an \(O(n \log n)\) time algorithm to compute \(|C_i|\), for \(i = 0, 1, \ldots, 10n\).
2. (18 points) Input are $k$ sets $S_1, S_2, \ldots, S_k$ such that $\sum_{i=1}^{k} |S_i| = n$. The problem is to determine if these sets are pairwise disjoint, i.e., $S_i \cap S_j = \emptyset$, for $i \neq j$. Show that $\Omega(n \log n)$ is a lower bound on the time needed to solve this problem.
3. (16 points) Prove or disprove:

If a problem π is in \( \mathcal{NP} \), then π can be solved using a deterministic algorithm that runs in time \( O(2^n) \), \( c \) being a constant.
4. (16 points) Let $G(V, E)$ be any graph. An independent set of $G$ is defined to be any subset $V'$ of $V$ such that each edge of $G$ is incident on at most one vertex in $V'$. The **Independent Set Problem** (ISP) takes as input a graph $G(V, E)$ and an integer $k \leq |V|$. The problem is to decide if $G$ has an independent set of size $k$. Is ISP in $\mathcal{P}$? If yes, present a polynomial time algorithm. If not, show that it is $\mathcal{NP}$-complete.
5. (15 points) Input is a graph $G(V,E)$ in adjacency matrix form. The problem is to determine if $G$ is directed or undirected. Present an $O(\log |V|)$ time algorithm for this problem. You can use up to $\frac{|V|^2}{\log |V|}$ CREW PRAM processors.
6. (20 points) The Linear Congruential Generator for generating pseudorandom integers uses the recurrence

\[ x_i = ax_{i-1} + b \pmod{n} \]

where \(a, b,\) and \(n\) are given integers, \(n\) being a prime. Also, \(x_1\) is given (and is known as the seed). Present an \(O(\log n)\)-time \(\frac{n}{\log n}\) processor CREW PRAM algorithm to compute the first \(n\) numbers from this sequence.