Fuzzy Information Granulation in Chains-of-Thought

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Abstract

In this paper, the author presents how Zadeh's fuzzy information granulation fits into the concept of chains-of-thought. Chains-of-thought, proposed by the author in [2], may be represented by using fuzzy graph structures. The author presents some earlier formalisms that have led to the development of chain-of-thought structures and relates these to Zadeh's ideas in computing with words [10] and fuzzy information granulation [11]. Further avenues of research are also presented.

INTRODUCTION

Mental cognitive processes are, literally, invisible and somewhat expressed in action under voluntary, intentional control. In [2], the author defines *cognitive diagnosis* as "the task of inferring the differences between a person's cognitive state and some desired or target state." This is fundamental in human cognition, such as in monitoring the problem solving performance of an individual in a tutoring environment. In such a scenario, an approximation of the individual's mental cognitive processes is required to correctly identify any misconceptions or bugs in the problem solving approach used [1]. Observations gathered also tend to pertain to partial, or complete, solutions proposed by the individual being observed.

Also in [2], the author defines *chains of thought* as "a string of cognitive states representing some aspect of an individual's thought processes." This, of course, pertains to the task under consideration. In situations where cognitive diagnosis may play a significant role; *e.g.* tutoring sessions, one must deal with chains of thought. Since the approximation of this structure is inherently fuzzy by nature, we investigate any implications with current work in computing with words [10] and fuzzy information granulation [11].

REVIEW OF LITERATURE

In [3], the author began investigating the role of cognitive diagnosis in the tutoring environment. It was proposed that a four-tiered system (see Table 1) may be needed to sufficiently model the process of approximating chains of thought. Perhaps the biggest point made by this requirement is the author's belief in structured concept representations [4] as indicated by Levels 0 and 1 of Table 1. Level 2 facilitates functionality by keeping track of instantiations of the more abstract concepts in Level 0. Level 3 determines the perceived behavior based on values associated to instances in Level 2. Hence, the higher levels represent observable qualities and lower levels represent deep knowledge.

TABLE 1. Underlying hierarchical organization for modeling cognitive diagnosis (from [2]).

Level	*Name*	*Attribute*
3	Behavioral	valuations
2	Functional	instantiations
1	Structural	relations
0	Conceptual	abstractions

Notice how the hierarchical organization of Table 1 relates to Zadeh's basic structure of fuzzy information granulation [11], which consists of granulation, attribution, and valuation as a whole is decomposed into parts. This suggests increasing specificity from granulation to valuation. Zadeh also points out the use of objects, (fuzzy) granules, (fuzzy) attributes, and (fuzzy) values for each of these levels. Objects and granules correspond to the abstractions and relations of Table 1, attributes to instantiations, and fuzzy values correspond to specific valuations impacting observable behavior.

TABLE 2. Zadeh's basic structure of fuzzy information granulation (from [12]).

Component	*Detail*
Granulation	object ? fuzzy granules
Attribution	granule ? fuzzy attributes
Valuation	attribute ? fuzzy values

As Zadeh points out in [11], "granulation is hierarchical in nature." Granulation offers a natural organization of the relationships between general and specific information in representations. These are important concepts to consider when formulating mental models.

In the next section, the author presents formulations for fuzzy chains of thought. These structures are proposed for use in cognitive diagnosis by a machine. In essence, the goal is to model the human cognitive capability of approximating a third person's chain of thought.

FUZZY CHAINS-OF-THOUGHT

To facilitate modeling of cognitive diagnosis by a machine in a tutoring environment, the author proposed the use of fuzzy chain of thought structures in [2]. These structures are derived from fuzzy cognitive maps, whose formulations also appear in [2].

A *fuzzy cognitive map* (or FCM) on a finite universe X is defined as a fuzzy graph identified by the 2-tuple $M = \langle C_M, R_M \rangle$ where:

- $C_M \in [0,1]^X$ is a fuzzy concept space of X; and
- R_M is a fuzzy multirelation on $C_M \in [0,1]^X$.

Structure preserving FCM homomorphisms, which consist of both node and path mappings, are used to derive an indication of the degree of similarity between FCMs.

For a fuzzy set $A \in [0,1]^X$ defined on a universe of discourse X, denote by $\mathbf{F}(A)$ the *fuzzy power set* of A. Then, a *chain of thought structure* on a finite universe X is a 5-tuple $S = \langle C, R, \Psi, \Phi, \delta \rangle$ where:

- C and R denote a fuzzy concept space and a fuzzy (multi)relational space for an FCM denoted by $\langle C, R \rangle$;
- Ψ and Φ are sets of sub-FCMs of $\langle C, R \rangle$ such that for $\langle C_\Psi, R_\Psi \rangle \in \Psi$ and $\langle C_\Phi, R_\Phi \rangle \in \Phi$ then $C_\Psi, C_\Phi \subseteq \mathbf{F}(C)$ and $R_\Psi, R_\Phi \subseteq \mathbf{F}(R)$; Ψ and Φ denote the knowledge state space and input space, respectively; and
- δ: $\Psi \times \Phi$? Ψ is a transition function for the chain of thought structure.

Structure preserving, transition preserving, and consistency preserving chain-of-thought homomorphisms are used to derive an indication of the degree of similarity between chain of thought structures.

The proposed methodology for cognitive diagnosis presented in [2], which takes the perspective of a tutor, requires a tri-map configuration consisting of a domain-specific FCM, a problem-specific FCM, and an (Approximated) Novice's FCM. The latter two have chains of thought associated with them: the tutor's "ideal" chain of thought and an "approximated" chain of thought derived from observation. Notice that FCMs fall into the Conceptual and Structural levels depicted in Table 1; chain of thought structures fall into the Functional and Behavioral levels.

IMPLICATIONS

During a tutoring session, a tutor approximates a novice's mental cognitive processes through discussions and by observing the novice's calculations, notes, and diagrams. Because of this, Zadeh's computing with words [10] and information granulation [11] have a natural connection with the task of cognitive diagnosis.

The rationale for computing with words, in this case, is more on "tolerance for imprecision, uncertainty, and partial truth" more so than "available information is not precise enough to justify the use of numbers" [11]. As exhibited by the underlying hierarchical organization for modeling cognitive diagnosis given in Table 1, fuzzy information granulation will play an essential role in further formulations for cognitive diagnosis by intelligent systems. Further, recall that in [11], the machinery of fuzzy information granulation includes fuzzy linguistic variables, fuzzy if-then rules, and fuzzy graphs. In actually implementing any proposed methodology to model cognitive diagnosis by a machine, fuzzy linguistic variables may be used to compute the terminal data set (TDS) from the initial data set (IDS). For example, in the formalism of chain of thought structures, the IDS may be represented by the "ideal" chain of thought structure, whereas the TDS may be represented by the "approximated" chain of thought structure. Further, fuzzy if-then rules may be used to infer a TDS from a IDS.

SUMMARY AND CONCLUSIONS

The panoply of tools presented in [1], [2], and [3] provide a good starting point for continued research in the area of cognitive diagnosis. In particular, the fuzzy cognitive maps and chain of thought structures in [2], along with their corresponding homomorphisms, furnish the necessary ground work for intelligent systems to conduct cognitive diagnosis.

The connections between fuzzy information granulation and the concept of chains-of-thought were presented by the author in this brief document. Since cognitive diagnosis is one of many tasks humans seem to be able to perform with ease, fuzzy information granulation should be integrated into any proposed methodology to model this task in an intelligent, artificial system.

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