

# Inference and Decision Making with Fuzzy Cognitive Structures

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*Abstract – In this paper, the author elaborates on the feasibility of a framework that uses fuzzy chain-of-thought structures. The emphasis is on how inference and decision making can be achieved using these cognitive structures. Applications in cognitive diagnosis, such as intelligent tutoring systems and cognitive modeling, are also discussed.*

Keywords: chain-of-thought, cognitive diagnosis, decision making, cognitive structures, inference.

## 1 Introduction

Cognitive diagnosis can be defined as “the task of inferring the differences between a person’s cognitive state and some desired or expected target cognitive state” [2]. This task is fundamental in human cognition, such as in monitoring the problem solving performance of an individual in a tutoring environment. In such a scenario, an approximation of the individual’s mental cognitive processes is required to correctly identify any misconceptions or bugs in the problem solving approach used by the novice [3]. Observations gathered also tend to pertain to partial, sometimes complete, solutions proposed by the individual being observed.

In addition, in [3], the author defines chains of thought as “a string of cognitive states representing some aspect of an individual’s thought processes.” This, of course, pertains to the task under consideration. In situations where cognitive diagnosis may play a significant role; e.g. tutoring sessions, one must deal with chains of thought. Since the approximation of this structure is inher-

ently fuzzy by nature, its connection and relevance to current work in *computing with words* [6] and *fuzzy information granulation* [7] have also been investigated [1].

## 2 Chains-of-Thought

The use of fuzzy chain-of-thought structures to model cognitive diagnosis in a tutoring environment was proposed in [3]. These structures are derived from fuzzy cognitive maps, whose formulations also appear in [2].

A *fuzzy cognitive map* (or FCM) on a finite universe  $X$  is defined as a fuzzy graph identified by the 2-tuple  $M = \langle C_M, R_M \rangle$  where:

- $C_M \in [0,1]^X$  is a fuzzy concept space of  $X$ ; and
- $R_M$  is a fuzzy multirelation on  $C_M \in [0,1]^X$ .

Structure preserving FCM homomorphisms, which consist of both node and path mappings (see [2]), derive a degree of similarity between FCMs.

For a fuzzy set  $A \in [0,1]^X$  defined on a universe of discourse  $X$ , denote by  $F(A)$  the *fuzzy power set* of  $A$ . Then, a *chain of thought structure* on a finite universe  $X$  is a 5-tuple  $S = \langle C, R, \Psi, \Phi, \delta \rangle$  where:

- $C$  and  $R$  denote a fuzzy concept space and a fuzzy (multi)relational space for an FCM denoted by  $\langle C, R \rangle$ ;
- $\Psi$  and  $\Phi$  are sets of sub-FCMs of  $\langle C, R \rangle$  such that for  $\langle C_\Psi, R_\Psi \rangle \in \Psi$  and  $\langle C_\Phi, R_\Phi \rangle \in \Phi$  then  $C_\Psi, C_\Phi \subseteq F(C)$  and  $R_\Psi, R_\Phi \subseteq F(R)$ ;  $\Psi$  and  $\Phi$  denote the knowledge state space and input space, respectively; and
- $\delta: \Psi \times \Phi \rightarrow \Psi$  is a transition function for the chain of thought structure.

Structure preserving, transition preserving, and consistency preserving chain-of-thought homomorphisms (see [2]) are used to derive a degree of similarity between chain-of-thought structures.

## 2.1 Framework

The framework for cognitive diagnosis in the tutoring environment outlined in [3] uses a four-tiered system (see Table 1), originally proposed in [4]. Levels 0 and 1 of Table 1 capture the essence of structured concept representations. Level 2 facilitates functionality by keeping track of instantiations of the more abstract concepts in Level 0. Level 3 determines the perceived behavior based on values associated to instances in Level 2. Hence, the higher levels represent observable qualities and lower levels represent deep knowledge.

Table 1. Hierarchical organization for modeling cognitive diagnosis (from [4]).

Level	Name	Attribute
3	Behavioral	valuations
2	Functional	instantiations
1	Structural	relations
0	Conceptual	abstractions

The hierarchical organization of Table 1 relates to Zadeh's basic structure of fuzzy information granulation [7], which consists of granulation, attribution, and valuation as a whole is decomposed into parts. It was suggested, in [1], that objects and granules correspond to the abstractions and relations of Table 1, attributes to instantiations, and fuzzy values correspond to specific valuations influencing observable behavior.

## 2.2 Models

The proposed methodology for cognitive diagnosis presented in [3], which takes the perspective of a tutor, requires a *tri-map configuration* of fuzzy cognitive maps (FCMs) consisting of a domain-specific FCM, a problem-specific FCM, and an (approximated) novice's FCM. (We shall refer to these three FCM structures as  $D$ ,  $P$ , and  $A$ , respectively.) Such an arrangement is required since the similarities and/or differences (see Figure 1 and [2,3] for a definition

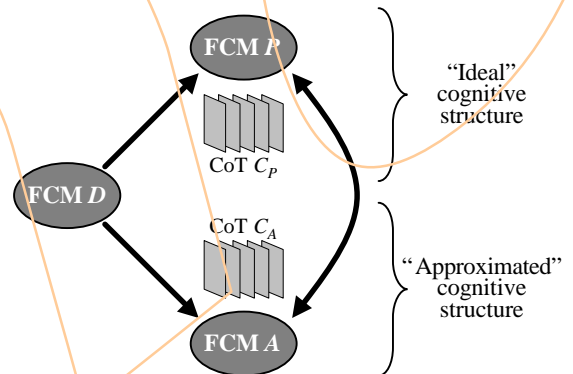


Figure 1. *Tri-map configuration* of FCMs [3].

of FCM operations) between the following pairs of FCMs can be used to derive useful information:

- $D - P$ , calculated between the domain-specific and problem-specific FCMs – identifies immediate concepts that are required to solve a particular problem.

- $P - A$  or  $A - P$ , calculated between the problem-specific and approximated FCMs – identifies either correct use of concepts or misconceptions the novice problem-solver may have in attempting to solve a particular problem.
- $D - A$ , calculated between the domain-specific and approximated FCMs – identifies concepts used by a novice that may or may not be related to solve a particular problem.

Furthermore, the  $P$  and  $A$  FCMs have fuzzy chain-of-thought (CoT) structures associated with them: the tutor’s “ideal” chain-of-thought,  $C_P$ , for the particular problem being considered, and an “approximated” chain-of-thought,  $C_A$ , derived from observation (see Figure 1). The chain-of-thought homomorphisms given in [3,4] identify similarities and/or differences between these structures.

Hence, FCMs model objects and granules (or the abstractions and relations, from Levels 0 and 1 in Table 1) via fuzzy graph structures, while fuzzy chain-of-thought structures model attributes and fuzzy values derived from observable behavior.

### 3 Inference

*Modus ponens* deduction is a tool for making inferences in rule-based systems. This is normally written as

$$\frac{\text{IF } x \text{ is } A \text{ THEN } y \text{ is } B.}{x \text{ is } A.} \quad y \text{ is } B. \quad (1)$$

The fuzzy extension to the above concept is *approximate reasoning*. Hence, for fuzzy sets  $A, A' \subseteq [0,1]^X$  and  $B, B' \subseteq [0,1]^Y$

$$\frac{\text{IF } x \text{ is } A \text{ THEN } y \text{ is } B.}{x \text{ is } A'.} \quad y \text{ is } B'. \quad (2)$$

where  $B'$  is calculated using *the compositional rule of inference*:

$$B' = A' \circ R(A,B). \quad (3)$$

In Equation (3) above,  $R(A,B)$  denotes an IF-THEN rule calculated as

$$(A \times B) \cup (A \times Y) \quad (4)$$

whose interpretation differs for the classical logic case in Equation (1) and the fuzzy logic case in Equation (2). The composition operator,  $\circ$ , in Equation (3) is usually interpreted as *max-min composition*.

Equations (2) through (4) form the basic building blocks for inference in fuzzy rule-based systems. Independent of the underlying framework used to model cognitive diagnosis, the fuzzy inference techniques may be used for the response analyzer, user status control, and diagnoser modules of an intelligent tutoring system (see, for example, [3,4]).

For example, using the framework discussed in this paper, a novice problem solver’s partial or complete solution to a problem may be represented as the FCM  $A$ . After calculating  $P - A$ , an intelligent tutoring system can be programmed to respond appropriately to the novice user. Additionally,  $A$  (and its corresponding fuzzy chain-of-thought structure) may be used to build a *student model*. Appropriate responses are generated based on the “status” of this model. These responses can be controlled by fuzzy rules similar to:

IF *the student model seems to indicate that the student user is doing good* BUT *this student user’s proposed solution is somewhat off*, THEN *give response R and advice S*.

Just like other intelligent systems, the fuzzy rule base used by response analyzer can be fine-tuned by adjusting the membership functions used, selecting another implication operator, etc.

## 4 Decision Making

Decisions made during cognitive diagnosis involve directing the diagnostic session. For example, with intelligent tutoring systems, decision making primarily pertains to directing the current tutoring session.

Most decisions are made based on a number of information sources and objectives. In a tutoring session, the direction of a tutoring session can be based on a number of factors. These may include the tutor's record of what problems the student has solved or attempted to solve, the tutor's internal student model, the tutor's sense of the student's rank relative to the rest of the class, and possibly many others.

Tutors also have alternatives to choose from when conducting a tutoring session. These alternatives may include sample problems to present to the student, various pedagogical approaches, and others. Selecting an alternative over another depends on the current goals and objectives of the tutor.

Fuzzy multiobjective decision making [5] involves the selection of one alternative,  $a_i$ , from a set of  $n$  alternatives  $A=\{a_1, \dots, a_n\}$  given a set of  $r$  objectives  $O=\{O_1, \dots, O_r\}$ . Denote by  $\mu_{O_j}(a_i)$  the degree to which alternative  $a_i$  satisfies the criteria specified for objective  $O_j$ . The decision function,  $D$ , satisfying all the decision objectives is

$$D = O_1 \cap O_2 \cap \dots \cap O_r \quad (5)$$

For each alternative,  $a$ , the grade of membership that the decision function,  $D$ , has is given by

$$\mu_D(a) = \min(\mu_{O_1}(a), \mu_{O_2}(a), \dots, \mu_{O_r}(a)) \quad (6)$$

Hence, the optimum decision,  $a^*$ , will be the alternative,  $a$ , that satisfies

$$\mu_D(a^*) = \max_{a \in A} (\mu_D(a)) \quad (7)$$

The significance of each of the tutoring objectives may be naturally specified through subjective information. This can be achieved by using a linear and ordinal set of preferences,  $P$ , so that each objective,  $O_j$ , will have a preference,  $p_j$ , associated with it [5]. Objectives and preferences are related by a decision measure,  $M(O_j, p_j)$ , that is typically defined as

$$\begin{aligned} M(O_j(a), p_j) &= p_j \rightarrow O_j(a) \\ &= \bar{p}_j \vee O_j(a) \end{aligned} \quad (8)$$

This changes the decision model given by Equation (5) to

$$D = \prod_{j=1}^r (\bar{p}_j \vee O_j(a)) \quad (9)$$

Hence, defining  $C_j = \bar{p}_j \cup O_j$ , Equation (7) for the optimum decision,  $a^*$ , changes to

$$\mu_D(a^*) = \max_{a \in A} (\min(\mu_{C_1}(a), \dots, \mu_{C_r}(a))) \quad (10)$$

## 5 Discussion

In this section, we briefly discuss inference and decision-making as presented in the previous section, relative to a particular proposed implementation of an intelligent tutoring system. This system is called *Diagnosis in Problem Solving* (or *DIPS*) [3,4]. In particular, the discussion will revolve around the *Diagnosis Module* of DIPS (see Figure 2).

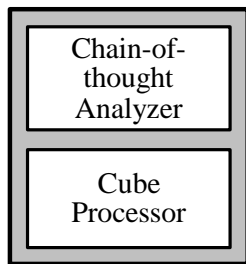


Figure 2. DIPS *Diagnosis Module* [3].

The DIPS Diagnosis Module consists of the *Chain-of-thought Analyzer* and the *Cube Processor*. (The concept of the “Cube” is not pertinent to this discussion and will not be elaborated in this paper.) The Chain-of-Thought Analyzer embodies all FCM and chain-of-thought structure operations, including all homomorphisms and the tri-map configuration (see Figure 1) discussed earlier. Pertinent results from these operations are passed to the Cube Processor via this submodule’s Communication Interface. The details of the submodules of the Cube Processor are given in Figure 3.

An area of cognitive diagnosis that fuzzy inference can be used is that of the *response analyzer* module (see Figure 3). In an intelligent tutoring system, this is the part of the system that takes “linguistic” input from the user. This input is converted to a representation similar to the FCM formalism. This converted information is used to calculate how far or near a user’s partial and/or complete solutions are from the “ideal” solution recognized by the system (denoted by  $P-A$  or  $A-P$  from Figure 1). A fuzzy rule base may then be used to formulate appropriate responses to the user.

Two other areas could benefit from the use of fuzzy inference in conjunction with the framework presented in this paper. The *user status control* module and the *diagnoser* module (see Figure 3) are these areas – both of which affect the student model. Again, a fuzzy rule base may be used to represent subjective information, from expert tutors, which are pertinent in updating a student

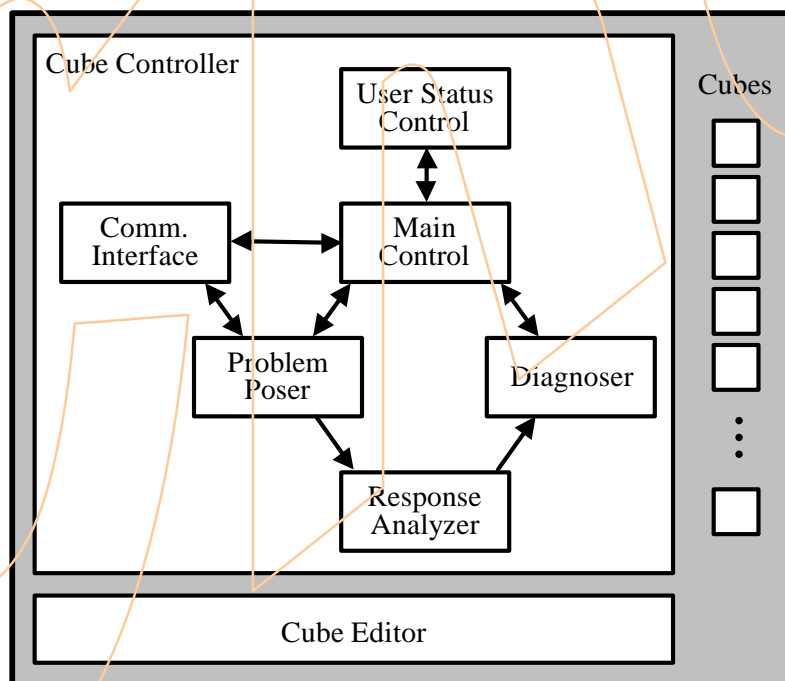


Figure 3. Submodules of the DIPS *Cube Processor* [3].

model. This information may also be used to allow the system to select from a variety of pedagogical styles (e.g. expert problem solvers require less verbose explanations than novice problem solver) as part of the decision-making responsibility of the system.

Notice that the response analyzer, user status control, and diagnoser modules are connected to the *main control* module in Figure 3. This implies that the main control module can be designed as a *subjective controller* – a controller that uses a fuzzy rule base to embody subjective knowledge.

Fuzzy orderings [3] may also be used for ordering problems in the *Cubes* (which are actually collections of problems that are somehow related to one another) and/or in ordering student models. This type of ordering information would be useful for multiobjective decision making for some of the submodules of the DIPS Cube Processor.

The *problem poser* module in DIPS's Cube Processor decides what problem(s) to present to the user. Based on subjective information from the current student model (provided by the *user status control* module) and the *diagnoser* module, fuzzy multiobjective decision making can be used to determine various, related courses of action that affect other parts of the Cube Processor.

Recall that both the objectives and the preferences are subjective in nature. Furthermore, preferences to objectives are typically adjusted to accommodate a particular situation. Such adjustments can be delegated to the main control module of the DIPS Cube Processor. This upgrades the main control to an intelligent controller that houses the objectives, the preferences associated to the objectives, and the fuzzy rule base (see Figure 4). With this arrangement, the main control module would better direct the problem

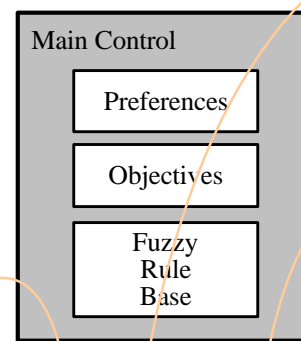


Figure 4. Components of an intelligent controller for cognitive diagnosis.

poser, user status control, and diagnoser modules of the Cube Processor. Fine-tuning of the controller would be facilitated by this design since its behavior can be modified just as any fuzzy controller is adjusted.

## 6 Summary and Conclusions

In this paper, the author provided details on the feasibility of a framework that uses fuzzy cognitive structures called fuzzy chains-of-thought. These structures also use fuzzy cognitive maps as a subcomponent. Inference and decision making with these cognitive structures were also discussed. Discussions focused on intelligent tutoring systems as a sample application in cognitive diagnosis. More importantly, the DIPS Cube Processor was used as an example illustrating how these cognitive structures can be used in implementing a diagnostic module (Figure 3) that uses an intelligent controller (Figure 4).

A major portion of the framework presented in this paper is currently being used in designing a web-based tutoring system for algorithm development and programming. It is projected that this tutoring system will illustrate the feasibility of using the proposed framework, in conjunction with fuzzy inference and fuzzy multiobjective decision making, to model cognitive diagnosis.

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