SOME BASIC FORMULATIONS FOR IMPLEMENTING A DIAGNOSTIC EXPERT SYSTEM BASED ON COGNITIVE MAPS

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Abstract

This paper discusses some of the basic theoretical formulations related to an ongoing exploratory study concerning the cognitive processes involved during scientific problem solving. It presents an approach we propose that utilizes conceptual graph structures called *fuzzy cognitive maps* to approximate the novice problem solver's current *chain–of–thought*. These structures provide a cognitive model of the novice. Set–theoretic characteristics suggest an effective fault–detection scheme based on overlays, while order relations are utilized for error diagnosis. Fuzzy formulations for these conceptual structures are also presented.

1 Introduction

This study is concerned with the development of an intelligent system for diagnosing problem solving skills. Initial efforts stem from related research in the fields of education and the physical sciences. A series of extensive studies in classical genetics problem solving [10,11] is the current major reference.

These studies have resulted in the proposal [12] of an overall design for a working expert system that takes the role of an expert tutor. It incorporates a *chain–of–thought analyser* that establishes a cognitive model of the student, to distinguish from student modeling approaches utilized by other existing intelligent tutoring systems (ITSs). In this paper, we present a solution that not only performs the diagnostic task; it suggests effective tutoring capabilities as well. We refer the reader to our preceding paper [6] that discusses the theoretical framework of our research efforts.

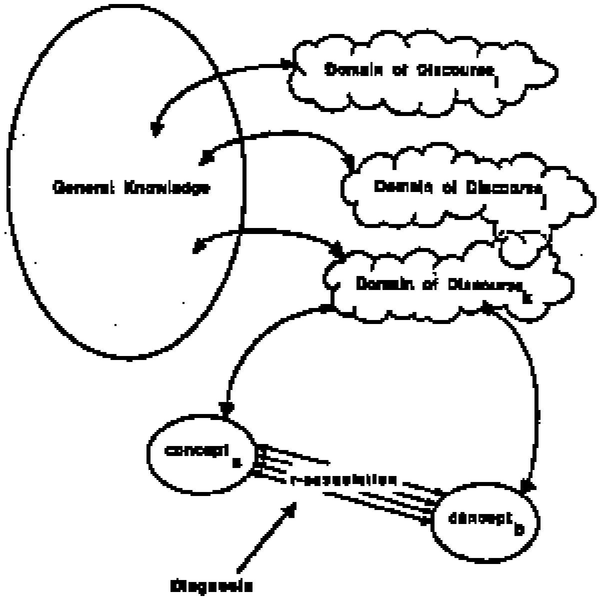

Figure 1: Preliminary notions

2 Fuzzy Cognitive Maps

The mathematical formulations presented here stem from graph theory, set theory, and relations theory. Some formulations are derived from Rosenfeld's *fuzzy graphs* [9] and from Kosko's *fuzzy cognitive maps* [8].

2.1 General Knowledge, Domains, and Concepts

Some preliminary notions are required before establishing a framework for cognitive maps designed for fault-detection and error diagnosis in scientific problem solving. The notion of *concepts* should be of primary interest in studies regarding conceptual perception of problems and their solutions. A *concept* is considered to be a set of *ideas* here.

The conjectured interrelationship between knowledge, domains and concepts is depicted in Figure 1. Mathematically, we refer to general knowledge as a *universe*, X. The domains of discourse are also known as *concept spaces*, $\mathcal{C} \in \mathcal{P}(X)$. $\mathcal{P}(X)$ denotes an applicable formulation for the *power set* of X, the set of all its subsets. Concepts constitute knowledge and understanding. Clearly, any concept $c \in X$ has the property that $c \in \mathcal{C}$, for at least one $\mathcal{C} \in \mathcal{P}(X)$. These initial conceptions are the basis for the mathematical formulations developed here.

2.2 Fuzzy Relations on a Fuzzy Set

The following definitions of fuzzy relations on a fuzzy set are adopted from [7,9,13]. Whereas these and other papers focus on *fuzzy binary relations*, which is also the concern in this study, some general definitions for n–*ary fuzzy relations* are outlined as the basis for succeeding formulations.

Let X be a set. A *fuzzy subset* $\tilde{S}$ of X is a set characterized by the mapping

$$\mu_{\tilde{S}}:\ X \rightarrow [0,1] \qquad (1)$$

which assigns to each element $x \in X$ a *membership degree*, $0 \leq \mu_{\tilde{S}}(x) \leq 1$. Another notation for a fuzzy subset $\tilde{S}$ of X is $\tilde{S} \in [0,1]^X$, where $[0,1]^X$ denotes the *fuzzy power set* of X — the set of all fuzzy subsets of X. Similarly, an n–*ary fuzzy relation* $\tilde{R}$ on X is a fuzzy subset of the product space X^n. $\tilde{R}$ is characterized by the mapping

$$\rho_{\tilde{R}}:\ X^n \rightarrow [0,1] \qquad (2)$$

which assigns to each n–tuple $(x_1, x_2, \ldots, x_n) \in X^n$ a *relational strength*, $0 \leq \rho_{\tilde{R}}(x_1, x_2, \ldots, x_n) \leq 1$, which is the degree of certainty that $\tilde{R}$ holds. In the special cases where μ and ρ can only take on the values 0 and 1, they reduce to the characteristic functions of a crisp subset of X and a crisp n–ary relation on X, respectively.

In general, given a fuzzy subset $\tilde{S}$ of X and an n–ary fuzzy relation $\tilde{R}$ on X, we refer to $\tilde{R}$ as a n–ary fuzzy relation on the fuzzy set $\tilde{S}$ if

$$\rho_{\tilde{R}}(x_1, x_2, \ldots, x_n) \ \leq\ \bigwedge_i \mu_{\tilde{S}}(x_i) \qquad (3)$$

where the $\bigwedge_i$ indicates the *minimum* with respect to $i = 1, \ldots, n$. Hence, the relational strength amongst elements of an n–tuple in $\tilde{R}$ is not allowed to exceed their membership degrees in $\tilde{S}$.

The symbols "$\nearrow$" and "$\searrow$" will be used to denote the *fuzzy extension* and *fuzzy restriction*, respectively, of a fuzzy mapping. Let $\tilde{F}$ be a fuzzy subset of a set X, defined as $\tilde{F}:\ X \rightarrow [0,1]$. $\tilde{R}$ is an n–ary fuzzy relation on $\tilde{F}$, characterized by the membership function ρ defined similarly to the fuzzy relation $\rho_{\tilde{R}}$ on $\tilde{S}$ above. The *fuzzy restriction* or *f–restriction* of $\tilde{R}$ on $\tilde{E} \subseteq \tilde{F}$, written $\tilde{R}\searrow_{\tilde{E}}$, is the fuzzy n–ary relation $\tilde{P}$ denoted by $\psi:\ \tilde{E}^n \rightarrow [0,1]$, with the property that

$$\psi(x_1, x_2, \ldots, x_n) \ =\ \rho(x_1, x_2, \ldots, x_n) \ \wedge\ \bigwedge_i \mu_{\tilde{E}}(x_i) \qquad (4)$$

for $i = 1, \ldots, n$. A *fuzzy extension* or *f–extension* of $\tilde{R}$ on $\tilde{G} \supseteq \tilde{F}$, written $\tilde{R}\nearrow_{\tilde{G}}$, is the fuzzy n–ary relation $\tilde{T}$ denoted by $\tau:\ \tilde{G}^n \rightarrow [0,1]$, with the property that

$$\bigwedge_i \mu_{\tilde{F}}(x_i) \ \leq\ \tau(x_1, x_2, \ldots, x_n) \ \leq\ \bigwedge_i \mu_{\tilde{G}}(x_i) \qquad (5)$$

for $i = 1, \ldots, n$. Notice that for all x in $\tilde{F}$, $\mu_{\tilde{E}}(x) \leq \mu_{\tilde{F}}(x) \leq \mu_{\tilde{G}}(x)$, and that $\tilde{T}$'s f–restriction to $\tilde{F}$ is $\tilde{R}$, written $\tilde{T}\searrow_{\tilde{F}} = \tilde{R}$.

We refer to the *minimal f–extension* whenever we consider fuzzy extensions on $\tilde{R}$. Relative to the definitions just outlined, the minimal f–extension of $\tilde{R}$ on $\tilde{G} \supseteq \tilde{F}$ is the fuzzy n–ary relation $\tilde{T}$ denoted by $\tau:\ \tilde{G}^n \rightarrow [0,1]$, and given by

$$\tau(x_1, x_2, \ldots, x_n) \ =\ \begin{cases} \rho(x_1, x_2, \ldots, x_n), & \text{if this is defined;} \\ 0, & \text{otherwise.} \end{cases} \qquad (6)$$

2.3 Definitions

Let the universe X be a non–empty finite set. X is called the *reference set* in some fuzzy literature [7]. The *degree of subsethood* of $\tilde{A}$ in $\tilde{B}$ for fuzzy subsets $\tilde{A}, \tilde{B} \in [0,1]^X$, written $\mu_{[0,1]^{\tilde{B}}}\tilde{A}$, is a measure of uncertainty in terms of the possibility, π, defined as

$$\mu_{[0,1]^{\tilde{B}}}\tilde{A} \ =\ \pi(\tilde{A} \subseteq \tilde{B}) \qquad (7)$$

where $\mu_{[0,1]^{\tilde{B}}}\tilde{A}$ denotes the *membership function* that maps $\tilde{A}$ to the power set of $\tilde{B}$. The value returned by $\mu_{[0,1]^{\tilde{B}}}\tilde{A}$ would depend on the *fuzzy implication operator* utilized to embody the said mapping, since by definition

$$\pi(\tilde{A} \subseteq \tilde{B}) \ =\ \min_{x \in X} (\mu_{\tilde{A}}x \rightarrow \mu_{\tilde{B}}x). \qquad (8)$$

This is similar to a *harsh criterion* for computing the *triangle subproduct* of two binary relations, given by

$$(R \triangleleft S)_{ik} \ =\ \min_j (R_{ij} \rightarrow S_{jk}) \qquad (9)$$

in relations theory. This latter equation derives the degree of subsethood of the *afterset* of R with the *foreset* of S for some fixed $x_i \in \mathrm{dom}(R)$ and $x_k \in \mathrm{ran}(S)$. A *moderate criterion* utilizes the *mean*.

We are now ready to define a *fuzzy cognitive map*, $\tilde{M} = (\mathcal{C}_{\tilde{M}}, \mathcal{R}_{\tilde{M}})$ over the universe X. Zadeh's fuzzy sets theory [13] utilizes the closed interval of reals $\Re = [0,1]$ as a *range set*, although linguistic terms are also possible. Hence, a *fuzzy cognitive map* is a *fuzzy graph* [9] that is a 2–tuple of functions $\mu:\ \mathcal{C} \rightarrow [0,1]$ and $\rho:\ \mathcal{C} \times \mathcal{C} \rightarrow [0,1]$, for $\mathcal{C} \in [0,1]^X$:

- $\mathcal{C}_{\tilde{M}} \in [0,1]^X$ is a *fuzzy concept space* of X. Unlike Kosko's formulations, the notions of *quantity space* and *modifier space* [8] are not considered here. These were developed in conjunction with the linguistically–inclined documentary encoding task of Axelrod.
- $\mathcal{R}_{\tilde{M}}$ is a fuzzy relation on $\mathcal{C}_{\tilde{M}} \in [0,1]^X$, given by $\mathcal{R}_{\tilde{M}}:\ \mathcal{C}_{\tilde{M}} \times \mathcal{C}_{\tilde{M}} \rightarrow \Re$. The values of $\mathcal{R}_{\tilde{M}}$ are restricted by formula (3), and so

  $$\mathcal{R}_{\tilde{M}ij} \leq \mu_{\mathcal{C}_{\tilde{M}}}x_i \wedge \mu_{\mathcal{C}_{\tilde{M}}}x_j$$

  for all $x_i, x_j \in \mathcal{C}_{\tilde{M}}$.
- The totally ordered range set $\Re = [0,1]$ establishes the *certainty* or *membership* values between concepts $C \in \mathcal{C}_{\tilde{M}}$.

2.3.1 Operations

We define the following operations on arbitrary fuzzy cognitive maps $\tilde{M} = (\mathcal{C}_{\tilde{M}}, \mathcal{R}_{\tilde{M}})$ and $\tilde{N} = (\mathcal{C}_{\tilde{N}}, \mathcal{R}_{\tilde{N}})$ over the universe X:

- The *fuzzy join* of $\tilde{M}$ and $\tilde{N}$ is denoted by

$$\tilde{M} \oplus \tilde{N} = (\mathcal{C}_{\tilde{M}} \cup \mathcal{C}_{\tilde{N}}, \mathcal{R}_{\tilde{M}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}}) \sqcup \mathcal{R}_{\tilde{N}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}})) \quad (10)$$

  where $\cup$ is the *fuzzy set union* operator and $\sqcup$ is the *fuzzy relational union* operator, defined as

$$\forall x \in X, \ \mu_{c_{\tilde{M}} \cup c_{\tilde{N}}} x = \mu_{c_{\tilde{M}}} x \vee \mu_{c_{\tilde{N}}} x$$
$$(\mathcal{R}_{\tilde{M}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}}) \sqcup \mathcal{R}_{\tilde{N}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}}))_{ij} = (\mathcal{R}_{\tilde{M}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}}))_{ij} \vee (\mathcal{R}_{\tilde{N}} \nearrow (c_{\tilde{M}} \cup c_{\tilde{N}}))_{ij}$$

  respectively. The $\vee$ symbol denotes the *maximum* operator.

- The *fuzzy meet* of $\tilde{M}$ and $\tilde{N}$ is denoted by

$$\tilde{M} \otimes \tilde{N} = (\mathcal{C}_{\tilde{M}} \cap \mathcal{C}_{\tilde{N}}, \mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}}) \sqcap \mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}})) \quad (11)$$

  where $\cap$ is the *fuzzy set intersection* operator and $\sqcap$ the *fuzzy relational intersection* operator, defined as

$$\forall x \in X, \ \mu_{c_{\tilde{M}} \cap c_{\tilde{N}}} x = \mu_{c_{\tilde{M}}} x \wedge \mu_{c_{\tilde{N}}} x$$
$$(\mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}}) \sqcap \mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}}))_{ij} = (\mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}}))_{ij} \wedge (\mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \cap c_{\tilde{N}}))_{ij}$$

  respectively. The $\wedge$ symbol denotes the *minimum* operator.

- The *fuzzy discrepancy* of $\tilde{M}$ over $\tilde{N}$ is denoted by

$$\tilde{M} \ominus \tilde{N} = (\mathcal{C}_{\tilde{M}} \setminus \mathcal{C}_{\tilde{N}}, \mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}) \sqcap \neg(\mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}))) \quad (12)$$

  where $\setminus$ is the *fuzzy set difference* operator and $\neg$ is the *fuzzy relational negation* operator, defined as

$$\forall x \in X, \ \mu_{c_{\tilde{M}} \setminus c_{\tilde{N}}} x = \mu_{c_{\tilde{M}}} x \wedge 1 - (\mu_{c_{\tilde{N}}} x)$$
$$(\mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}) \sqcap \mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}))_{ij} = (\mathcal{R}_{\tilde{M}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}))_{ij} \wedge (\mathcal{R}_{\tilde{N}} \searrow (c_{\tilde{M}} \setminus c_{\tilde{N}}))_{ij}$$

  respectively.

The fuzzy join operation can be utilised to create and link cognitive maps. Truncation or pruning is accomplished by taking the fuzzy meet. Meanwhile, fault-detection is performed by taking the fuzzy discrepancy of the approximated chain-of-thought with a cognitive map supplied by the expert.

2.3.2 Relations

The following relations on arbitrary fuzzy cognitive maps $\tilde{M} = (\mathcal{C}_{\tilde{M}}, \mathcal{R}_{\tilde{M}})$ and $\tilde{N} = (\mathcal{C}_{\tilde{N}}, \mathcal{R}_{\tilde{N}})$ over the universe X are defined:

- $\tilde{M}$ is called a *fuzzy submap* of $\tilde{N}$, written $\tilde{M} \lessdot \tilde{N}$, if the following hold:

$$\tilde{M} \lessdot \tilde{N} \Leftrightarrow \pi(\mathcal{C}_{\tilde{M}} \subseteq \mathcal{C}_{\tilde{N}}) \geq \alpha_{\subseteq}, \text{ and} \quad (13)$$
$$\mathcal{R}_{\tilde{M}} \sqsubseteq \mathcal{R}_{\tilde{N}} \searrow c_{\tilde{M}} \quad (14)$$

  for some preset threshold $\alpha_{\subseteq}$, usually set at $\alpha_{\subseteq} = 0.5$, and where

$$\pi(\mathcal{C}_{\tilde{M}} \subseteq \mathcal{C}_{\tilde{N}}) = \min_{x \in X} (\mu_{c_{\tilde{M}}} x \rightarrow \mu_{c_{\tilde{N}}} x) \text{ and}$$
$$\mathcal{R}_{\tilde{M}} \sqsubseteq \mathcal{R}_{\tilde{N}} \searrow c_{\tilde{M}} \Leftrightarrow \forall i,j \ \ \mathcal{R}_{\tilde{M}ij} \leq (\mathcal{R}_{\tilde{N}} \searrow c_{\tilde{M}})_{ij}.$$

- $\tilde{M}$ is called a *fuzzy supermap* of $\tilde{N}$, written $\tilde{M} \gtrdot \tilde{N}$, if the following hold:

$$\tilde{M} \gtrdot \tilde{N} \Leftrightarrow \pi(\mathcal{C}_{\tilde{M}} \supseteq \mathcal{C}_{\tilde{N}}) \geq \alpha_{\supseteq}, \text{ and} \quad (15)$$
$$\mathcal{R}_{\tilde{M}} \searrow c_{\tilde{N}} \sqsupseteq \mathcal{R}_{\tilde{N}} \quad (16)$$

  for some preset threshold $\alpha_{\supseteq}$, usually set at $\alpha_{\supseteq} = 0.5$, and where

$$\pi(\mathcal{C}_{\tilde{M}} \supseteq \mathcal{C}_{\tilde{N}}) = \min_{x \in X} (\mu_{c_{\tilde{N}}} x \rightarrow \mu_{c_{\tilde{M}}} x) \text{ and}$$
$$\mathcal{R}_{\tilde{M}} \searrow c_{\tilde{N}} \sqsupseteq \mathcal{R}_{\tilde{N}} \Leftrightarrow \forall i,j \ \ (\mathcal{R}_{\tilde{M}} \searrow c_{\tilde{N}})_{ij} \geq \mathcal{R}_{\tilde{N}ij}.$$

- $\tilde{M}$ is said to be *equal* to $\tilde{N}$, written $\tilde{M} = \tilde{N}$, when the following hold:

$$\tilde{M} = \tilde{N} \Leftrightarrow \mathcal{C}_{\tilde{M}} = \mathcal{C}_{\tilde{N}} \text{ and } \mathcal{R}_{\tilde{M}} = \mathcal{R}_{\tilde{N}} \quad (17)$$

  which means that

$$\forall x \in X, \ \mu_{c_{\tilde{M}}} x = \mu_{c_{\tilde{N}}} x$$
$$\text{and } \forall i,j \ \mathcal{R}_{\tilde{M}ij} = \mathcal{R}_{\tilde{N}ij}.$$

  Otherwise, $\tilde{M}$ is *not equal* to $\tilde{N}$, written $\tilde{M} \neq \tilde{N}$.

The cognitive map relations just presented could be utilised for comparing and analysing approximated structures with idealised ones. Fuzzy orders [14] could provide interpretations of conceptual manipulation strategies employed by a particular novice. Other useful applications are still being investigated.

3 Implementation: Some Useful Properties

Some of the relevant knowledge acquisition problems are alleviated by utilising the concept of *closures* from mathematical relations theory [3,4]. This allows the free-hand entry of data by connecting frame-like nodal structures representing concepts. The system will then attempt to establish the desired relation and conceptual ordering prior to analysis.

It is conjectured here that for any cognitive map $M = (\mathcal{C}_M, \mathcal{R}_M)$, $\mathcal{R}_M$ is the *transitive closure* of the entered nodal connectivity, $\mathcal{R}'$, supplied during the knowledge acquisition phase. This stems from the proposal that $\mathcal{R}_M$ is *transitive*. Choose to consider $\mathcal{R}'$ *irreflexive* and *strictly anti-symmetric*, since in *adjacency matrix* notation, the properties

$$\forall r \in \mathcal{R}, \quad \mathcal{R}'_{ii} \leq r, \qquad (18)$$
$$\text{and} \quad \min_{i,j}(\mathcal{R}'_{ij}, \mathcal{R}'_{ji}) = 0 \qquad (19)$$

hold. The *transitive closure* of $\mathcal{R}'$, which is the least inclusive transitive relation containing $\mathcal{R}'$ [4], is a *strict order* that is exactly $\mathcal{R}_M$. This relation is computed by taking the *circlet product* or the max–min composition of the *local preorder closure* of $\mathcal{R}'$ with itself

$$\text{tra clo } \mathcal{R}' = (\text{locpre clo } \mathcal{R}') \circ \mathcal{R}' = \mathcal{R}_M \qquad (20)$$

as suggested by a fast algorithm in [4].

4 Conclusions

We have proposed the use of fuzzy cognitive maps to represent an approximation of the concept manipulation and formation strategy employed by a novice in attempting to solve a particular problem. Corresponding fuzzy formulations are developed utilising relations theory and the notion of restrictions and extensions. Interpretation of the resulting conceptual graph structures, which would be parallel to a conceived *solution path*, is derived by utilizing order relations. Furthermore, this approach's applicability to other abductive systems requiring some fault detection or error diagnosis is being considered. We are currently investigating other plausible possibilistic formulations as well. These provide sufficient initiative to continue extensive research on the subject area.

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